31 RADIOACTIVITY AND NUCLEAR PHYSICS

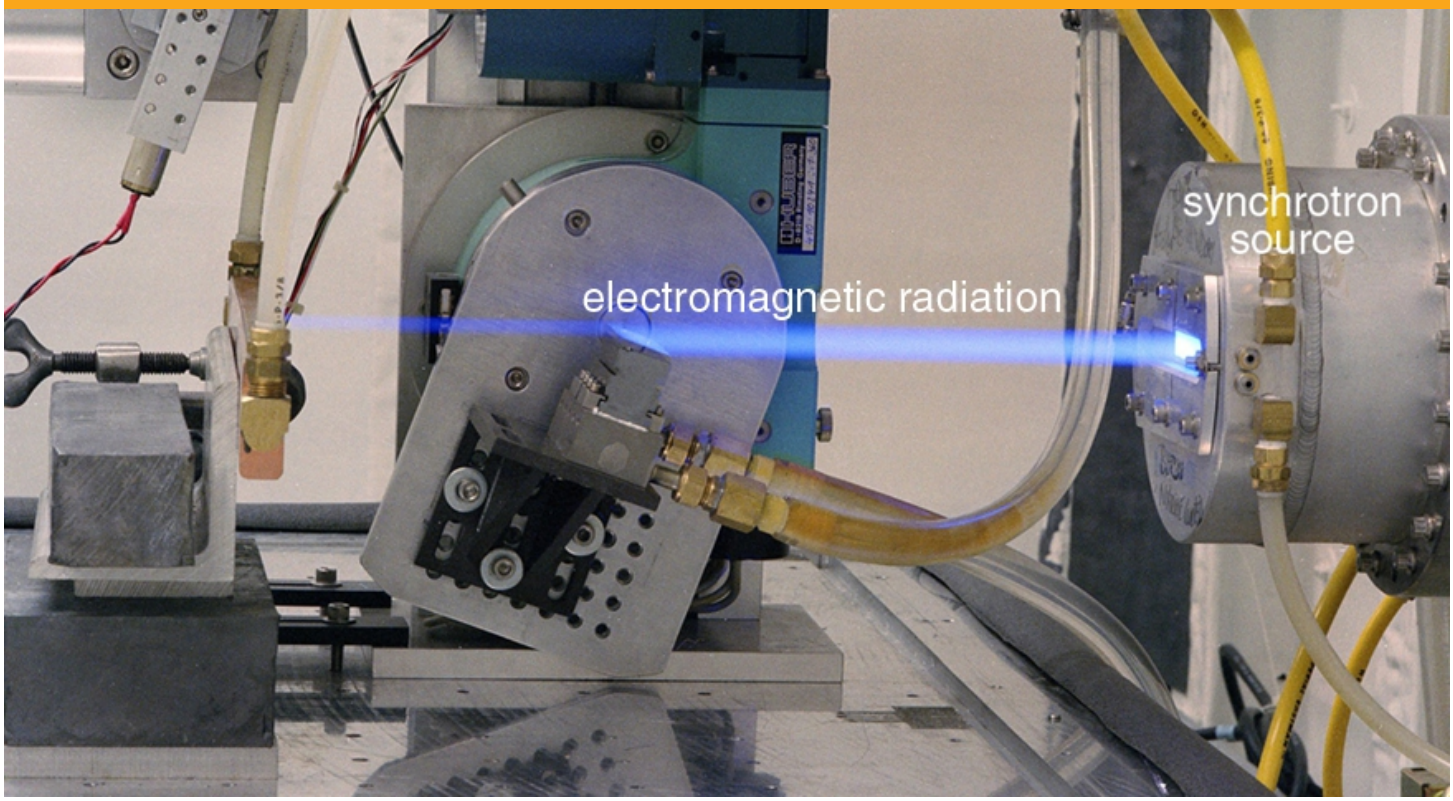


Figure 31.1 The synchrotron source produces electromagnetic radiation, as evident from the visible glow. (credit: United States Department of Energy, via Wikimedia Commons)

Learning Objectives

Introduction to Radioactivity and Nuclear Physics

There is an ongoing quest to find substructures of matter. At one time, it was thought that atoms would be the ultimate substructure, but just when the first direct evidence of atoms was obtained, it became clear that they have a substructure and a tiny *nucleus*. The nucleus itself has spectacular characteristics. For example, certain nuclei are unstable, and their decay emits radiations with energies millions of times greater than atomic energies. Some of the mysteries of nature, such as why the core of the earth remains molten and how the sun produces its energy, are explained by nuclear phenomena. The exploration of *radioactivity* and the nucleus revealed fundamental and previously unknown particles, forces, and conservation laws. That exploration has evolved into a search for further underlying structures, such as quarks. In this chapter, the fundamentals of nuclear radioactivity and the nucleus are explored. The following two chapters explore the more important applications of nuclear physics in the field of medicine. We will also explore the basics of what we know about quarks and other substructures smaller than nuclei.

31.1 Nuclear Radioactivity

The discovery and study of nuclear radioactivity quickly revealed evidence of revolutionary new physics. In addition, uses for nuclear radiation also emerged quickly—for example, people such as Ernest Rutherford used it to determine the size of the nucleus and devices were painted with radon-doped paint to make them glow in the dark (see **Figure 31.2**). We therefore begin our study of nuclear physics with the discovery and basic features of nuclear radioactivity.

Figure 31.2 The dials of this World War II aircraft glow in the dark, because they are painted with radium-doped phosphorescent paint. It is a poignant reminder of the dual nature of radiation. Although radium paint dials are conveniently visible day and night, they emit radon, a radioactive gas that is hazardous and is not directly sensed. (credit: U.S. Air Force Photo)

Discovery of Nuclear Radioactivity

In 1896, the French physicist Antoine Henri Becquerel (1852–1908) accidentally found that a uranium-rich mineral called pitchblende emits invisible, penetrating rays that can darken a photographic plate enclosed in an opaque envelope. The rays therefore carry energy; but amazingly, the pitchblende emits them continuously without any energy input. This is an apparent violation of the law of conservation of energy, one that we now understand is due to the conversion of a small amount of mass into energy, as related in Einstein's famous equation $E = mc^2$. It was soon evident that Becquerel's rays originate in the nuclei of the atoms and have other unique characteristics. The emission of these rays is called **nuclear radioactivity** or simply **radioactivity**. The rays themselves are called **nuclear radiation**. A nucleus that spontaneously destroys part of its mass to emit radiation is said to **decay** (a term also used to describe the emission of radiation by atoms in excited states). A substance or object that emits nuclear radiation is said to be **radioactive**.

Two types of experimental evidence imply that Becquerel's rays originate deep in the heart (or nucleus) of an atom. First, the radiation is found to be associated with certain elements, such as uranium. Radiation does not vary with chemical state—that is, uranium is radioactive whether it is in the form of an element or compound. In addition, radiation does not vary with temperature, pressure, or ionization state of the uranium atom. Since all of these factors affect electrons in an atom, the radiation cannot come from electron transitions, as atomic spectra do. The huge energy emitted during each event is the second piece of evidence that the radiation cannot be atomic. Nuclear radiation has energies of the order of $10^6\ \text{eV}$ per event, which is much greater than the typical atomic energies (a few eV), such as that observed in spectra and chemical reactions, and more than ten times as high as the most energetic characteristic x rays. Becquerel did not vigorously pursue his discovery for very long. In 1898, Marie Curie (1867–1934), then a graduate student married the already well-known French physicist Pierre Curie (1859–1906), began her doctoral study of Becquerel's rays. She and her husband soon discovered two new radioactive elements, which she named *polonium* (after her native land) and *radium* (because it radiates). These two new elements filled holes in the periodic table and, further, displayed much higher levels of radioactivity per gram of material than uranium. Over a period of four years, working under poor conditions and spending their own funds, the Curies processed more than a ton of uranium ore to isolate a gram of radium salt. Radium became highly sought after, because it was about two million times as radioactive as uranium. Curie's radium salt glowed visibly from the radiation that took its toll on them and other unaware researchers. Shortly after completing her Ph.D., both Curies and Becquerel shared the 1903 Nobel Prize in physics for their work on radioactivity. Pierre was killed in a horse cart accident in 1906, but Marie continued her study of radioactivity for nearly 30 more years. Awarded the 1911 Nobel Prize in chemistry for her discovery of two new elements, she remains the only person to win Nobel Prizes in physics and chemistry. Marie's radioactive fingerprints on some pages of her notebooks can still expose film, and she suffered from radiation-induced lesions. She died of leukemia likely caused by radiation, but she was active in research almost until her death in 1934. The following year, her daughter and son-in-law, Irene and Frederic Joliot-Curie, were awarded the Nobel Prize in chemistry for their discovery of artificially induced radiation, adding to a remarkable family legacy.

Alpha, Beta, and Gamma

Research begun by people such as New Zealander Ernest Rutherford soon after the discovery of nuclear radiation indicated that different types of rays are emitted. Eventually, three types were distinguished and named **alpha** (α), **beta** (β), and **gamma** (γ), because, like x-rays, their identities were initially unknown. **Figure 31.3** shows what happens if the rays are passed through a magnetic field. The γ s are unaffected, while the α s and β s are deflected in opposite directions, indicating the α s are positive, the β s negative, and the γ s uncharged. Rutherford used both magnetic and electric fields to show that α s have a positive charge twice the magnitude of an electron, or $+2\,|\,q_e\,|$. In the process, he found the α s charge to mass ratio to be several thousand times smaller than the electron's. Later on, Rutherford collected α s from a radioactive source and passed an electric discharge through them, obtaining the spectrum of recently discovered helium gas. Among many important discoveries made by Rutherford and his collaborators was the proof that α *radiation is the emission of a helium nucleus*. Rutherford won the Nobel Prize in chemistry in 1908 for his early work. He continued to make important contributions until his death in 1934.

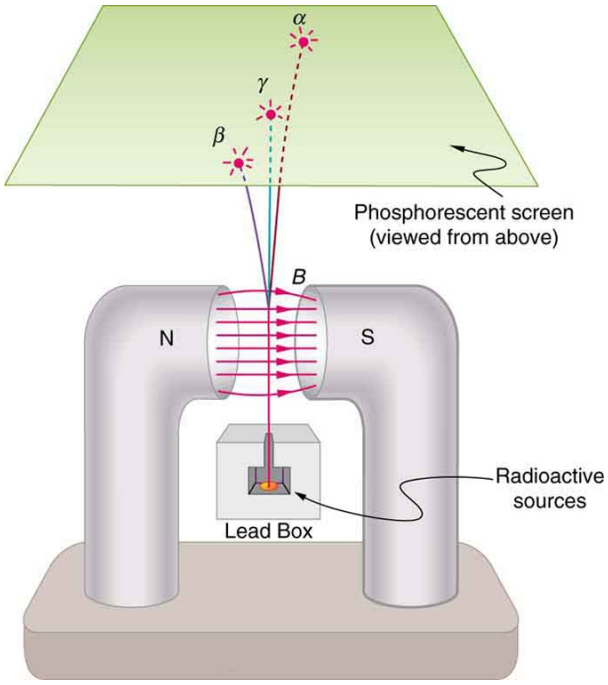


Figure 31.3 Alpha, beta, and gamma rays are passed through a magnetic field on the way to a phosphorescent screen. The α s and β s bend in opposite directions, while the γ s are unaffected, indicating a positive charge for α s, negative for β s, and neutral for γ s. Consistent results are obtained with electric fields. Collection of the radiation offers further confirmation from the direct measurement of excess charge.

Other researchers had already proved that β s are negative and have the same mass and same charge-to-mass ratio as the recently discovered electron. By 1902, it was recognized that β *radiation is the emission of an electron*. Although β s are electrons, they do not exist in the nucleus before it decays and are not ejected atomic electrons—the electron is created in the nucleus at the instant of decay.

Since γ s remain unaffected by electric and magnetic fields, it is natural to think they might be photons. Evidence for this grew, but it was not until 1914 that this was proved by Rutherford and collaborators. By scattering γ radiation from a crystal and observing interference, they demonstrated that γ *radiation is the emission of a high-energy photon by a nucleus*. In fact, γ radiation comes from the de-excitation of a nucleus, just as an x ray comes from the de-excitation of an atom. The names "γ ray" and "x ray" identify the source of the radiation. At the same energy, γ rays and x rays are otherwise identical.

Table 31.1 Properties of Nuclear Radiation

Type of Radiation	Range
α-Particles	A sheet of paper, a few cm of air, fractions of a mm of tissue
β-Particles	A thin aluminum plate, or tens of cm of tissue
γ Rays	Several cm of lead or meters of concrete

Ionization and Range

Two of the most important characteristics of α, β, and γ rays were recognized very early. All three types of nuclear radiation produce *ionization* in materials, but they penetrate different distances in materials—that is, they have different *ranges*. Let us examine why they have these characteristics and what are some of the consequences.

Like x rays, nuclear radiation in the form of α s, β s, and γ s has enough energy per event to ionize atoms and molecules in any material. The energy emitted in various nuclear decays ranges from a few keV to more than $10\ \text{MeV}$, while only a few eV are needed to produce ionization. The effects of x rays and nuclear radiation on biological tissues and other materials, such as solid state electronics, are directly related to the ionization they produce. All of them, for example, can damage electronics or kill cancer cells. In addition, methods for detecting x rays and nuclear radiation are based on ionization, directly or indirectly. All of them can ionize the air between the plates of a capacitor, for example, causing it to discharge. This is the basis of inexpensive personal radiation monitors, such as pictured in **Figure 31.4**. Apart from α, β, and γ, there are other forms of nuclear radiation as well, and these also produce ionization with similar effects. We define **ionizing radiation** as any form of radiation that produces ionization whether nuclear in origin or not, since the effects and detection of the radiation are related to ionization.

Figure 31.4 These dosimeters (literally, dose meters) are personal radiation monitors that detect the amount of radiation by the discharge of a rechargeable internal capacitor. The amount of discharge is related to the amount of ionizing radiation encountered, a measurement of dose. One dosimeter is shown in the charger. Its scale is read through an eyepiece on the top. (credit: L. Chang, Wikimedia Commons)

The **range of radiation** is defined to be the distance it can travel through a material. Range is related to several factors, including the energy of the radiation, the material encountered, and the type of radiation (see **Figure 31.5**). The higher the *energy*, the greater the range, all other factors being the same. This makes good sense, since radiation loses its energy in materials primarily by producing ionization in them, and each ionization of an atom or a molecule requires energy that is removed from the radiation. The amount of ionization is, thus, directly proportional to the energy of the particle of radiation, as is its range.

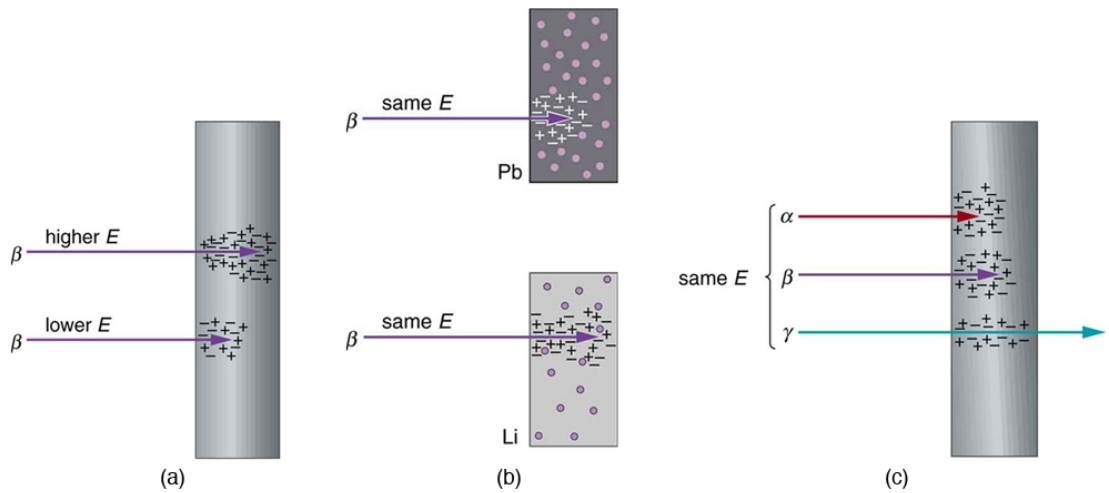


Figure 31.5 The penetration or range of radiation depends on its energy, the material it encounters, and the type of radiation. (a) Greater energy means greater range. (b) Radiation has a smaller range in materials with high electron density. (c) Alphas have the smallest range, betas have a greater range, and gammas penetrate the farthest.

Radiation can be absorbed or shielded by materials, such as the lead aprons dentists drape on us when taking x rays. Lead is a particularly effective shield compared with other materials, such as plastic or air. How does the range of radiation depend on *material*? Ionizing radiation interacts best with charged particles in a material. Since electrons have small masses, they most readily absorb the energy of the radiation in collisions. The greater the density of a material and, in particular, the greater the density of electrons within a material, the smaller the range of radiation.

Collisions

Conservation of energy and momentum often results in energy transfer to a less massive object in a collision. This was discussed in detail in **Work, Energy, and Energy Resources**, for example.

Different *types* of radiation have different ranges when compared at the same energy and in the same material. Alphas have the shortest range, betas penetrate farther, and gammas have the greatest range. This is directly related to charge and speed of the particle or type of radiation. At a given energy, each α, β, or γ will produce the same number of ionizations in a material (each ionization requires a certain amount of energy on average). The more readily the particle produces ionization, the more quickly it will lose its energy. The effect of *charge* is as follows: The α has a charge of $+2q_e$, the β has a charge of $-q_e$, and the γ is uncharged. The electromagnetic force exerted by the α is thus twice as strong as that exerted by the β and it is more likely to produce ionization. Although chargeless, the γ does interact weakly because it is an electromagnetic wave, but it is less likely to produce ionization in any encounter. More quantitatively, the change in momentum Δp given to a particle in the material is $\Delta p = F\Delta t$, where F is the force the α, β, or γ exerts over a time Δt. The smaller the charge, the smaller is F and the smaller is the momentum (and energy) lost. Since the speed of alphas is about 5% to 10% of the speed of light, classical (non-relativistic) formulas apply.

The *speed* at which they travel is the other major factor affecting the range of α s, β s, and γ s. The faster they move, the less time they spend in the vicinity of an atom or a molecule, and the less likely they are to interact. Since α s and β s are particles with mass (helium nuclei and electrons, respectively), their energy is kinetic, given classically by $\frac{1}{2}mv^2$. The mass of the β particle is thousands of times less than that of the α s, so that

β s must travel much faster than α s to have the same energy. Since β s move faster (most at relativistic speeds), they have less time to interact than α s. Gamma rays are photons, which must travel at the speed of light. They are even less likely to interact than a β, since they spend even less time near a given atom (and they have no charge). The range of γ s is thus greater than the range of β s.

Alpha radiation from radioactive sources has a range much less than a millimeter of biological tissues, usually not enough to even penetrate the dead layers of our skin. On the other hand, the same α radiation can penetrate a few centimeters of air, so mere distance from a source prevents α radiation from reaching us. This makes α radiation relatively safe for our body compared to β and γ radiation. Typical β radiation can penetrate a few millimeters of tissue or about a meter of air. Beta radiation is thus hazardous even when not ingested. The range of β s in lead is about a millimeter, and so it is easy to store β sources in lead radiation-proof containers. Gamma rays have a much greater range than either α s or β s. In fact, if a given thickness of material, like a lead brick, absorbs 90% of the γ s, then a second lead brick will only absorb 90% of what got through the first. Thus, γ s do not have a well-defined range; we can only cut down the amount that gets through. Typically, γ s can penetrate many meters of air, go right through our bodies, and are effectively shielded (that is, reduced in intensity to acceptable levels) by many centimeters of lead. One benefit of γ s is that they can be used as radioactive tracers (see **Figure 31.6**).

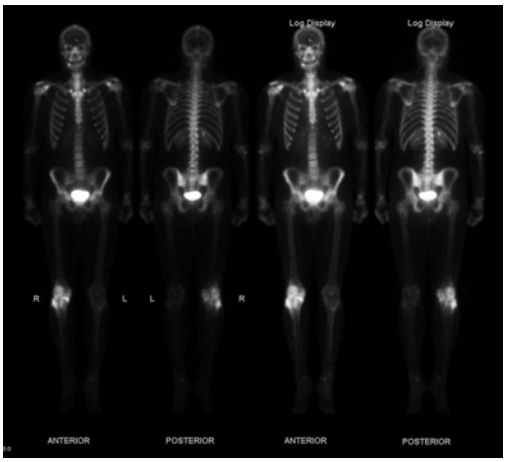


Figure 31.6 This image of the concentration of a radioactive tracer in a patient's body reveals where the most active bone cells are, an indication of bone cancer. A short-lived radioactive substance that locates itself selectively is given to the patient, and the radiation is measured with an external detector. The emitted γ radiation has a sufficient range to leave the body—the range of α s and β s is too small for them to be observed outside the patient. (credit: Kieran Maher, Wikimedia Commons)

PhET Explorations: Beta Decay

Watch beta decay occur for a collection of nuclei or for an individual nucleus.

PhET Interactive Simulation

Figure 31.7 Beta Decay (http://cnx.org/content/m42623/1.7/beta-decay_en.jar)

31.2 Radiation Detection and Detectors

It is well known that ionizing radiation affects us but does not trigger nerve impulses. Newspapers carry stories about unsuspecting victims of radiation poisoning who fall ill with radiation sickness, such as burns and blood count changes, but who never felt the radiation directly. This makes the detection of radiation by instruments more than an important research tool. This section is a brief overview of radiation detection and some of its applications.

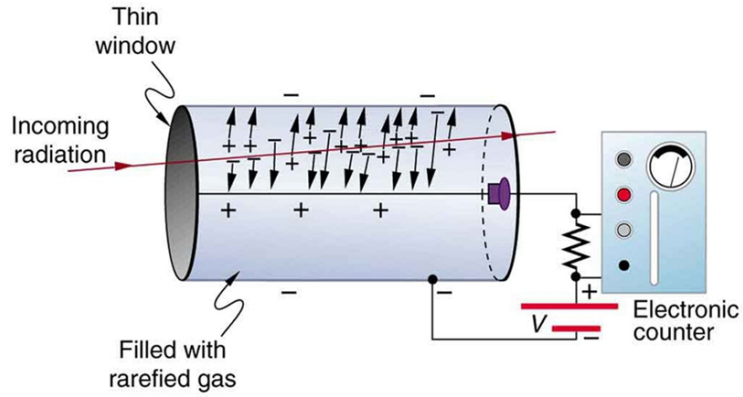
Human Application

The first direct detection of radiation was Becquerel's fogged photographic plate. Photographic film is still the most common detector of ionizing radiation, being used routinely in medical and dental x rays. Nuclear radiation is also captured on film, such as seen in **Figure 31.8**. The mechanism for film exposure by ionizing radiation is similar to that by photons. A quantum of energy interacts with the emulsion and alters it chemically, thus exposing the film. The quantum come from an α-particle, β-particle, or photon, provided it has more than the few eV of energy needed to induce the chemical change (as does all ionizing radiation). The process is not 100% efficient, since not all incident radiation interacts and not all interactions produce the chemical change. The amount of film darkening is related to exposure, but the darkening also depends on the type of radiation, so that absorbers and other devices must be used to obtain energy, charge, and particle-identification information.

Figure 31.8 Film badges contain film similar to that used in this dental x-ray film and is sandwiched between various absorbers to determine the penetrating ability of the radiation as well as the amount. (credit: Werneuchen, Wikimedia Commons)

Another very common **radiation detector** is the **Geiger tube**. The clicking and buzzing sound we hear in dramatizations and documentaries, as well as in our own physics labs, is usually an audio output of events detected by a Geiger counter. These relatively inexpensive radiation detectors are based on the simple and sturdy Geiger tube, shown schematically in **Figure 31.9**(b). A conducting cylinder with a wire along its axis is filled with an insulating gas so that a voltage applied between the cylinder and wire produces almost no current. Ionizing radiation passing through the tube produces free ion pairs that are attracted to the wire and cylinder, forming a current that is detected as a count. The word count implies that there is no information on energy, charge, or type of radiation with a simple Geiger counter. They do not detect every particle, since some radiation can pass through without producing enough ionization to be detected. However, Geiger counters are very useful in producing a prompt output that reveals the existence and relative intensity of ionizing radiation.

(a)



(b)

Figure 31.9 (a) Geiger counters such as this one are used for prompt monitoring of radiation levels, generally giving only relative intensity and not identifying the type or energy of the radiation. (credit: TimVickers, Wikimedia Commons) (b) Voltage applied between the cylinder and wire in a Geiger tube causes ions and electrons produced by radiation passing through the gas-filled cylinder to move towards them. The resulting current is detected and registered as a count.

Another radiation detection method records light produced when radiation interacts with materials. The energy of the radiation is sufficient to excite atoms in a material that may fluoresce, such as the phosphor used by Rutherford's group. Materials called **scintillators** use a more complex collaborative process to convert radiation energy into light. Scintillators may be liquid or solid, and they can be very efficient. Their light output can provide information about the energy, charge, and type of radiation. Scintillator light flashes are very brief in duration, enabling the detection of a huge number of particles in short periods of time. Scintillator detectors are used in a variety of research and diagnostic applications. Among these are the detection by satellite-mounted equipment of the radiation from distant galaxies, the analysis of radiation from a person indicating body burdens, and the detection of exotic particles in accelerator laboratories.

Light from a scintillator is converted into electrical signals by devices such as the **photomultiplier** tube shown schematically in **Figure 31.10**. These tubes are based on the photoelectric effect, which is multiplied in stages into a cascade of electrons, hence the name photomultiplier. Light entering the photomultiplier strikes a metal plate, ejecting an electron that is attracted by a positive potential difference to the next plate, giving it enough energy to eject two or more electrons, and so on. The final output current can be made proportional to the energy of the light entering the tube, which is in turn proportional to the energy deposited in the scintillator. Very sophisticated information can be obtained with scintillators, including energy, charge, particle identification, direction of motion, and so on.

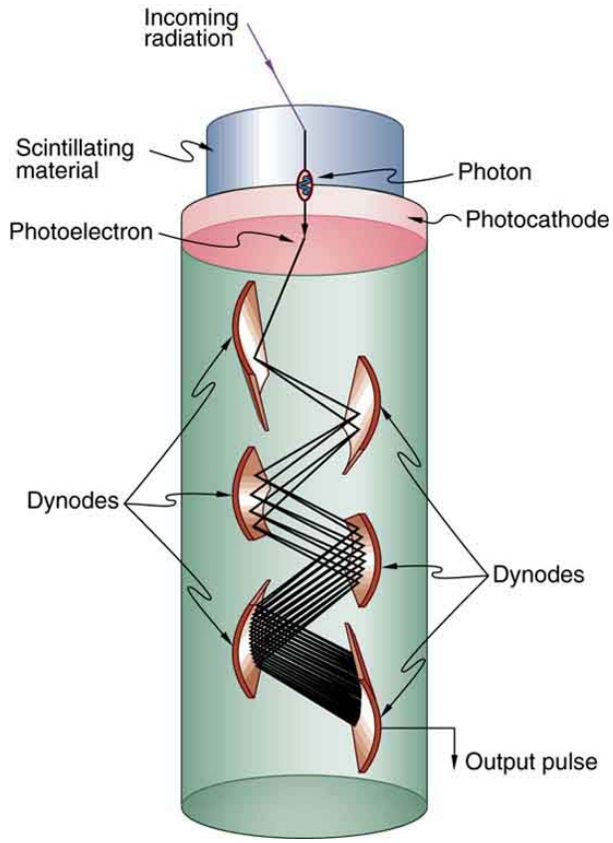


Figure 31.10 Photomultipliers use the photoelectric effect on the photocathode to convert the light output of a scintillator into an electrical signal. Each successive dynode has a more-positive potential than the last and attracts the ejected electrons, giving them more energy. The number of electrons is thus multiplied at each dynode, resulting in an easily detected output current.

Solid-state radiation detectors convert ionization produced in a semiconductor (like those found in computer chips) directly into an electrical signal. Semiconductors can be constructed that do not conduct current in one particular direction. When a voltage is applied in that direction, current flows only when ionization is produced by radiation, similar to what happens in a Geiger tube. Further, the amount of current in a solid-state detector is closely related to the energy deposited and, since the detector is solid, it can have a high efficiency (since ionizing radiation is stopped in a shorter distance in solids fewer particles escape detection). As with scintillators, very sophisticated information can be obtained from solid-state detectors.

PhET Explorations: Radioactive Dating Game

Learn about different types of radiometric dating, such as carbon dating. Understand how decay and half life work to enable radiometric dating to work. Play a game that tests your ability to match the percentage of the dating element that remains to the age of the object.

PhET Interactive Simulation

Figure 31.11 Radioactive Dating Game (http://cnx.org/content/m42627/1.4/radioactive-dating-game_en.jar)

31.3 Substructure of the Nucleus

What is inside the nucleus? Why are some nuclei stable while others decay? (See **Figure 31.12**.) Why are there different types of decay (α , β and γ)? Why are nuclear decay energies so large? Pursuing natural questions like these has led to far more fundamental discoveries than you might imagine.

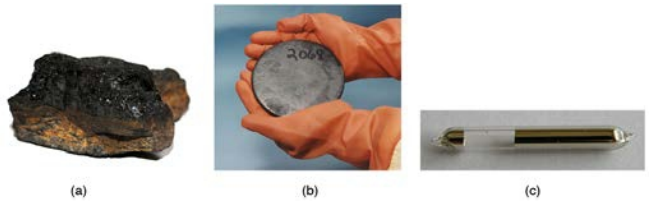


Figure 31.12 Why is most of the carbon in this coal stable (a), while the uranium in the disk (b) slowly decays over billions of years? Why is cesium in this ampule (c) even less stable than the uranium, decaying in far less than 1/1,000,000 the time? What is the reason uranium and cesium undergo different types of decay (α and β , respectively)? (credits: (a) Bresson Thomas, Wikimedia Commons; (b) U.S. Department of Energy; (c) Tomihahndorf, Wikimedia Commons)

We have already identified **protons** as the particles that carry positive charge in the nuclei. However, there are actually *two* types of particles in the nuclei—the *proton* and the *neutron*, referred to collectively as **nucleons**, the constituents of nuclei. As its name implies, the **neutron** is a neutral particle ($q = 0$) that has nearly the same mass and intrinsic spin as the proton. **Table 31.2** compares the masses of protons, neutrons, and electrons. Note how close the proton and neutron masses are, but the neutron is slightly more massive once you look past the third digit. Both nucleons are much more massive than an electron. In fact, $m_p = 1836 m_e$ (as noted in **Medical Applications of Nuclear Physics** and $m_n = 1839 m_e$.

Table 31.2 also gives masses in terms of mass units that are more convenient than kilograms on the atomic and nuclear scale. The first of these is the *unified* **atomic mass** *unit* (u), defined as

$$1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg}. \tag{31.1}$$

This unit is defined so that a neutral carbon ^{12}C atom has a mass of exactly 12 u. Masses are also expressed in units of MeV/c^2 . These units are very convenient when considering the conversion of mass into energy (and vice versa), as is so prominent in nuclear processes. Using $E = mc^2$ and units of m in MeV/c^2 , we find that c^2 cancels and E comes out conveniently in MeV. For example, if the rest mass of a proton is converted entirely into energy, then

$$E = mc^2 = (938.27 \text{ MeV}/c^2)c^2 = 938.27 \text{ MeV}. \tag{31.2}$$

It is useful to note that 1 u of mass converted to energy produces 931.5 MeV, or

$$1 \text{ u} = 931.5 \text{ MeV}/c^2. \tag{31.3}$$

All properties of a nucleus are determined by the number of protons and neutrons it has. A specific combination of protons and neutrons is called a **nuclide** and is a unique nucleus. The following notation is used to represent a particular nuclide:

$$^{A}_{Z}\text{X}_N, \tag{31.4}$$

where the symbols A , X , Z , and N are defined as follows: The *number of protons in a nucleus* is the **atomic number** Z , as defined in **Medical Applications of Nuclear Physics**. X is the *symbol for the element*, such as Ca for calcium. However, once Z is known, the element is known; hence, Z and X are redundant. For example, $Z = 20$ is always calcium, and calcium always has $Z = 20$. N is the *number of neutrons* in a nucleus. In the notation for a nuclide, the subscript N is usually omitted. The symbol A is defined as the number of nucleons or the *total number of protons and neutrons*,

$$A = N + Z, \tag{31.5}$$

where A is also called the **mass number**. This name for A is logical; the mass of an atom is nearly equal to the mass of its nucleus, since electrons have so little mass. The mass of the nucleus turns out to be nearly equal to the sum of the masses of the protons and neutrons in it, which is proportional to A . In this context, it is particularly convenient to express masses in units of u. Both protons and neutrons have masses close to 1 u, and so the mass of an atom is close to A u. For example, in an oxygen nucleus with eight protons and eight neutrons, $A = 16$, and its mass is 16 u. As noticed, the unified atomic mass unit is defined so that a neutral carbon atom (actually a ^{12}C atom) has a mass of *exactly* 12 u . Carbon was chosen as the standard, partly because of its importance in organic chemistry (see **Appendix A**).

Table 31.2 Masses of the Proton, Neutron, and Electron

Particle	Symbol	kg	u	$\text{MeV}c^2$
Proton	*p*	1.67262×10^{-27}	1.007276	938.27
Neutron	*n*	1.67493×10^{-27}	1.008665	939.57
Electron	*e*	9.1094×10^{-31}	0.00054858	0.511

Let us look at a few examples of nuclides expressed in the $^{A}_{Z}\text{X}_N$ notation. The nucleus of the simplest atom, hydrogen, is a single proton, or $^{1}_{1}\text{H}$ (the zero for no neutrons is often omitted). To check this symbol, refer to the periodic table—you see that the atomic number Z of hydrogen is 1.

Since you are given that there are no neutrons, the mass number A is also 1. Suppose you are told that the helium nucleus or α particle has two protons and two neutrons. You can then see that it is written ${}^{4}_{2}\mathrm{He}_2$. There is a scarce form of hydrogen found in nature called deuterium; its nucleus has one proton and one neutron and, hence, twice the mass of common hydrogen. The symbol for deuterium is, thus, ${}^{2}_{1}\mathrm{H}_1$ (sometimes D is used, as for deuterated water $\mathrm{D}_2\mathrm{O}$). An even rarer—and radioactive—form of hydrogen is called tritium, since it has a single proton and two neutrons, and it is written ${}^{3}_{1}\mathrm{H}_2$. These three varieties of hydrogen have nearly identical chemistries, but the nuclei differ greatly in mass, stability, and other characteristics. Nuclei (such as those of hydrogen) having the same Z and different N s are defined to be **isotopes** of the same element.

There is some redundancy in the symbols A, X, Z, and N. If the element X is known, then Z can be found in a periodic table and is always the same for a given element. If both A and X are known, then N can also be determined (first find Z; then, $N = A - Z$). Thus the simpler notation for nuclides is

$$^{A}\mathrm{X}, \tag{31.6}$$

which is sufficient and is most commonly used. For example, in this simpler notation, the three isotopes of hydrogen are ${}^{1}\mathrm{H}$, ${}^{2}\mathrm{H}$, and ${}^{3}\mathrm{H}$, while the α particle is ${}^{4}\mathrm{He}$. We read this backward, saying helium-4 for ${}^{4}\mathrm{He}$, or uranium-238 for ${}^{238}\mathrm{U}$. So for ${}^{238}\mathrm{U}$, should we need to know, we can determine that $Z = 92$ for uranium from the periodic table, and, thus, $N = 238 - 92 = 146$.

A variety of experiments indicate that a nucleus behaves something like a tightly packed ball of nucleons, as illustrated in **Figure 31.13**. These nucleons have large kinetic energies and, thus, move rapidly in very close contact. Nucleons can be separated by a large force, such as in a collision with another nucleus, but resist strongly being pushed closer together. The most compelling evidence that nucleons are closely packed in a nucleus is that the **radius of a nucleus**, r, is found to be given approximately by

$$r = r_0 A^{1/3}, \tag{31.7}$$

where $r_0 = 1.2\ \mathrm{fm}$ and A is the mass number of the nucleus. Note that $r^3 \propto A$. Since many nuclei are spherical, and the volume of a sphere is $V = (4/3)\pi r^3$, we see that $V \propto A$ —that is, the volume of a nucleus is proportional to the number of nucleons in it. This is what would happen if you pack nucleons so closely that there is no empty space between them.

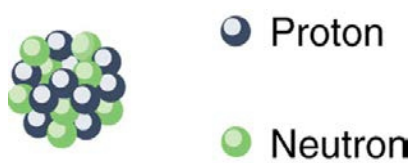


Figure 31.13 A model of the nucleus.

Nucleons are held together by nuclear forces and resist both being pulled apart and pushed inside one another. The volume of the nucleus is the sum of the volumes of the nucleons in it, here shown in different colors to represent protons and neutrons.

Example 31.1 How Small and Dense Is a Nucleus?

(a) Find the radius of an iron-56 nucleus. (b) Find its approximate density in kg/m^3, approximating the mass of ${}^{56}\mathrm{Fe}$ to be 56 u.

Strategy and Concept

(a) Finding the radius of ${}^{56}\mathrm{Fe}$ is a straightforward application of $r = r_0 A^{1/3}$, given $A = 56$. (b) To find the approximate density, we assume the nucleus is spherical (this one actually is), calculate its volume using the radius found in part (a), and then find its density from $\rho = m/V$. Finally, we will need to convert density from units of u/fm^3 to kg/m^3.

Solution

(a) The radius of a nucleus is given by

$$r = r_0 A^{1/3}. \tag{31.8}$$

Substituting the values for r_0 and A yields

$$\begin{aligned} r &= (1.2\ \mathrm{fm})(56)^{1/3} = (1.2\ \mathrm{fm})(3.83) \\ &= 4.6\ \mathrm{fm}. \end{aligned} \tag{31.9}$$

(b) Density is defined to be $\rho = m/V$, which for a sphere of radius r is

$$\rho = \frac{m}{V} = \frac{m}{(4/3)\pi r^3}. \tag{31.10}$$

Substituting known values gives

$$\begin{aligned}\rho &= \frac{56\text{ u}}{(1.33)(3.14)(4.6\text{ fm})^3} \\ &= 0.138\text{ u/fm}^3.\end{aligned} \tag{31.11}$$

Converting to units of kg/m^3, we find

$$\begin{aligned}\rho &= (0.138\text{ u/fm}^3)(1.66\times10^{-27}\text{ kg/u})\left(\frac{1\text{ fm}}{10^{-15}\text{ m}}\right) \\ &= 2.3\times10^{17}\text{ kg/m}^3.\end{aligned} \tag{31.12}$$

Discussion

(a) The radius of this medium-sized nucleus is found to be approximately 4.6 fm, and so its diameter is about 10 fm, or 10^{-14} m. In our discussion of Rutherford's discovery of the nucleus, we noticed that it is about 10^{-15} m in diameter (which is for lighter nuclei), consistent with this result to an order of magnitude. The nucleus is much smaller in diameter than the typical atom, which has a diameter of the order of 10^{-10} m.

(b) The density found here is so large as to cause disbelief. It is consistent with earlier discussions we have had about the nucleus being very small and containing nearly all of the mass of the atom. Nuclear densities, such as found here, are about 2×10^{14} times greater than that of water, which has a density of "only" 10^3 kg/m^3. One cubic meter of nuclear matter, such as found in a neutron star, has the same mass as a cube of water 61 km on a side.

Nuclear Forces and Stability

What forces hold a nucleus together? The nucleus is very small and its protons, being positive, exert tremendous repulsive forces on one another. (The Coulomb force increases as charges get closer, since it is proportional to $1/r^2$, even at the tiny distances found in nuclei.) The answer is that two previously unknown forces hold the nucleus together and make it into a tightly packed ball of nucleons. These forces are called the *weak and strong nuclear forces*. Nuclear forces are so short ranged that they fall to zero strength when nucleons are separated by only a few fm. However, like glue, they are strongly attracted when the nucleons get close to one another. The strong nuclear force is about 100 times more attractive than the repulsive EM force, easily holding the nucleons together. Nuclear forces become extremely repulsive if the nucleons get too close, making nucleons strongly resist being pushed inside one another, something like ball bearings.

The fact that nuclear forces are very strong is responsible for the very large energies emitted in nuclear decay. During decay, the forces do work, and since work is force times the distance ($W = Fd\cos\theta$), a large force can result in a large emitted energy. In fact, we know that there are *two* distinct nuclear forces because of the different types of nuclear decay—the strong nuclear force is responsible for α decay, while the weak nuclear force is responsible for β decay.

The many stable and unstable nuclei we have explored, and the hundreds we have not discussed, can be arranged in a table called the **chart of the nuclides**, a simplified version of which is shown in **Figure 31.14**. Nuclides are located on a plot of N versus Z. Examination of a detailed chart of the nuclides reveals patterns in the characteristics of nuclei, such as stability, abundance, and types of decay, analogous to but more complex than the systematics in the periodic table of the elements.

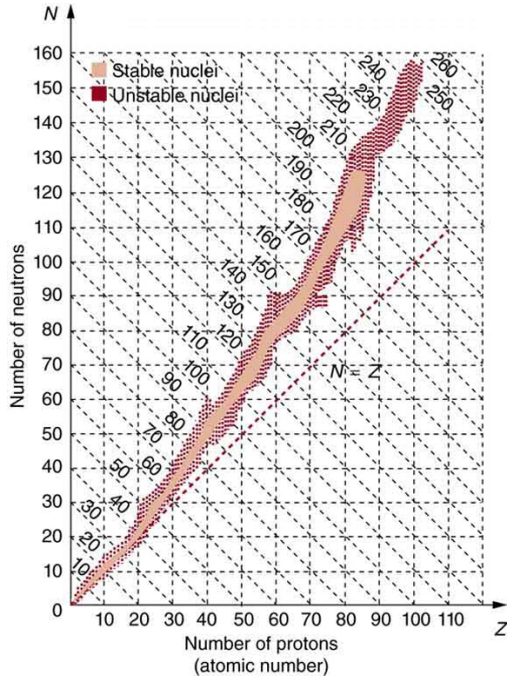


Figure 31.14 Simplified chart of the nuclides, a graph of N versus Z for known nuclides. The patterns of stable and unstable nuclides reveal characteristics of the nuclear forces. The dashed line is for $N = Z$. Numbers along diagonals are mass numbers A.

In principle, a nucleus can have any combination of protons and neutrons, but **Figure 31.14** shows a definite pattern for those that are stable. For low-mass nuclei, there is a strong tendency for N and Z to be nearly equal. This means that the nuclear force is more attractive when $N = Z$. More detailed examination reveals greater stability when N and Z are even numbers—nuclear forces are more attractive when neutrons and protons are in pairs. For increasingly higher masses, there are progressively more neutrons than protons in stable nuclei. This is due to the ever-growing repulsion between protons. Since nuclear forces are short ranged, and the Coulomb force is long ranged, an excess of neutrons keeps the protons a little farther apart, reducing Coulomb repulsion. Decay modes of nuclides out of the region of stability consistently produce nuclides closer to the region of stability. There are more stable nuclei having certain numbers of protons and neutrons, called **magic numbers**. Magic numbers indicate a shell structure for the nucleus in which closed shells are more stable. Nuclear shell theory has been very successful in explaining nuclear energy levels, nuclear decay, and the greater stability of nuclei with closed shells. We have been producing ever-heavier transuranic elements since the early 1940s, and we have now produced the element with $Z = 118$. There are theoretical predictions of an island of relative stability for nuclei with such high Z s.

Figure 31.15 The German-born American physicist Maria Goeppert Mayer (1906–1972) shared the 1963 Nobel Prize in physics with J. Jensen for the creation of the nuclear shell model. This successful nuclear model has nucleons filling shells analogous to electron shells in atoms. It was inspired by patterns observed in nuclear properties. (credit: Nobel Foundation via Wikimedia Commons)

31.4 Nuclear Decay and Conservation Laws

Nuclear **decay** has provided an amazing window into the realm of the very small. Nuclear decay gave the first indication of the connection between mass and energy, and it revealed the existence of two of the four basic forces in nature. In this section, we explore the major modes of nuclear decay; and, like those who first explored them, we will discover evidence of previously unknown particles and conservation laws.

Some nuclides are stable, apparently living forever. Unstable nuclides decay (that is, they are radioactive), eventually producing a stable nuclide after many decays. We call the original nuclide the **parent** and its decay products the **daughters**. Some radioactive nuclides decay in a single step to a stable nucleus. For example, ^{60}Co is unstable and decays directly to ^{60}Ni, which is stable. Others, such as ^{238}U, decay to another unstable nuclide, resulting in a **decay series** in which each subsequent nuclide decays until a stable nuclide is finally produced. The decay series that starts from ^{238}U is of particular interest, since it produces the radioactive isotopes ^{226}Ra and ^{210}Po, which the Curies first discovered (see **Figure 31.16**). Radon gas is also produced (^{222}Rn in the series), an increasingly recognized naturally occurring hazard. Since radon is a noble gas, it emanates from materials, such as soil, containing even trace amounts of ^{238}U and can be inhaled. The decay of radon and its daughters produces internal damage. The ^{238}U decay series ends with ^{206}Pb, a stable isotope of lead.

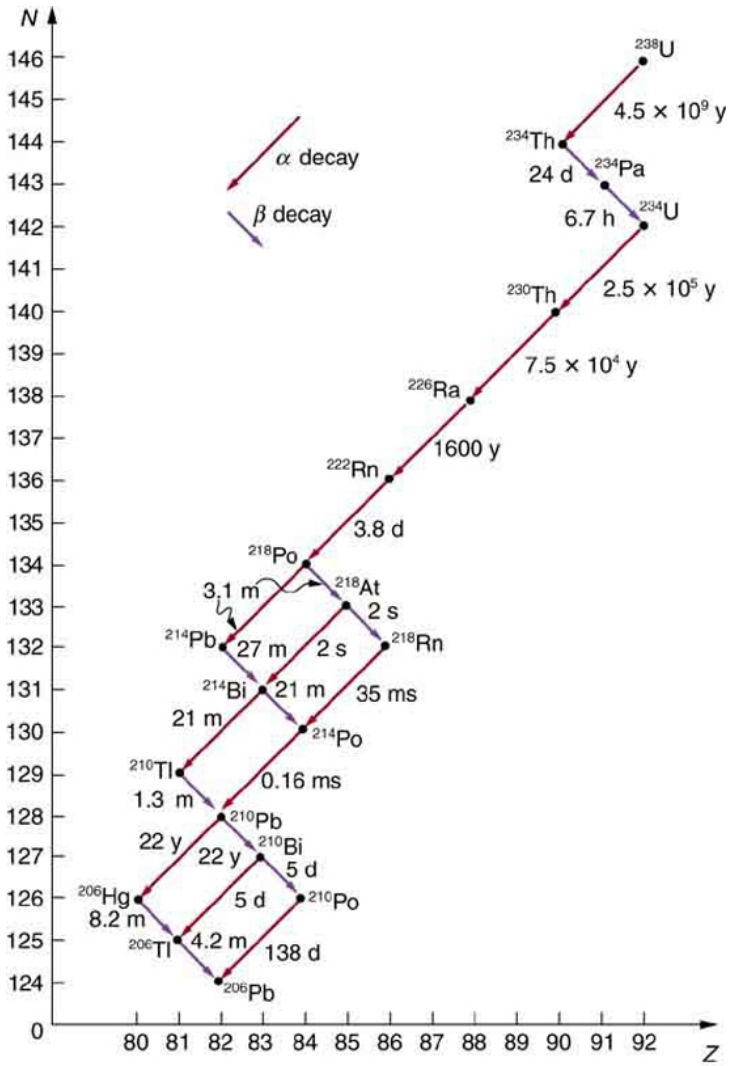


Figure 31.16 The decay series produced by ^{238}U, the most common uranium isotope. Nuclides are graphed in the same manner as in the chart of nuclides. The type of decay for each member of the series is shown, as well as the half-lives. Note that some nuclides decay by more than one mode. You can see why radium and polonium are found in uranium ore. A stable isotope of lead is the end product of the series.

Note that the daughters of α decay shown in **Figure 31.16** always have two fewer protons and two fewer neutrons than the parent. This seems reasonable, since we know that α decay is the emission of a ^{4}He nucleus, which has two protons and two neutrons. The daughters of β decay have one less neutron and one more proton than their parent. Beta decay is a little more subtle, as we shall see. No γ decays are shown in the figure, because they do not produce a daughter that differs from the parent.

Alpha Decay

In **alpha decay**, a ^{4}He nucleus simply breaks away from the parent nucleus, leaving a daughter with two fewer protons and two fewer neutrons than the parent (see **Figure 31.17**). One example of α decay is shown in **Figure 31.16** for ^{238}U. Another nuclide that undergoes α decay is ^{239}Pu. The decay equations for these two nuclides are

$$^{238}\text{U} \rightarrow {}^{234}\text{Th}_{92}^{234} + {}^{4}\text{He} \tag{31.13}$$

and

$$^{239}\text{Pu} \rightarrow {}^{235}\text{U} + {}^{4}\text{He}. \tag{31.14}$$

Figure 31.17 Alpha decay is the separation of a ^{4}He nucleus from the parent. The daughter nucleus has two fewer protons and two fewer neutrons than the parent. Alpha decay occurs spontaneously only if the daughter and ^{4}He nucleus have less total mass than the parent.

If you examine the periodic table of the elements, you will find that Th has $Z = 90$, two fewer than U, which has $Z = 92$. Similarly, in the second **decay equation**, we see that U has two fewer protons than Pu, which has $Z = 94$. The general rule for α decay is best written in the format $^{A}_{Z}\text{X}_{N}$. If a certain nuclide is known to α decay (generally this information must be looked up in a table of isotopes, such as in **Appendix B**), its α **decay equation** is

$$^{A}_{Z}\text{X}_{N} \rightarrow {}^{A-4}_{Z-2}\text{Y}_{N-2} + {}^{4}_{2}\text{He}_{2} \quad (\alpha \text{ decay}) \tag{31.15}$$

where Y is the nuclide that has two fewer protons than X, such as Th having two fewer than U. So if you were told that ^{239}Pu α decays and were asked to write the complete decay equation, you would first look up which element has two fewer protons (an atomic number two lower) and find that this is uranium. Then since four nucleons have broken away from the original 239, its atomic mass would be 235.

It is instructive to examine conservation laws related to α decay. You can see from the equation $^{A}_{Z}\text{X}_{N} \rightarrow {}^{A-4}_{Z-2}\text{Y}_{N-2} + {}^{4}_{2}\text{He}_{2}$ that total charge is conserved. Linear and angular momentum are conserved, too. Although conserved angular momentum is not of great consequence in this type of decay, conservation of linear momentum has interesting consequences. If the nucleus is at rest when it decays, its momentum is zero. In that case, the fragments must fly in opposite directions with equal-magnitude momenta so that total momentum remains zero. This results in the α particle carrying away most of the energy, as a bullet from a heavy rifle carries away most of the energy of the powder burned to shoot it. Total mass–energy is also conserved: the energy produced in the decay comes from conversion of a fraction of the original mass. As discussed in **Atomic Physics**, the general relationship is

$$E = (\Delta m)c^{2}. \tag{31.16}$$

Here, E is the **nuclear reaction energy** (the reaction can be nuclear decay or any other reaction), and Δm is the difference in mass between initial and final products. When the final products have less total mass, Δm is positive, and the reaction releases energy (is exothermic). When the products have greater total mass, the reaction is endothermic (Δm is negative) and must be induced with an energy input. For α decay to be spontaneous, the decay products must have smaller mass than the parent.

Example 31.2 Alpha Decay Energy Found from Nuclear Masses

Find the energy emitted in the α decay of ^{239}Pu.

Strategy

Nuclear reaction energy, such as released in α decay, can be found using the equation $E = (\Delta m)c^{2}$. We must first find Δm, the difference in mass between the parent nucleus and the products of the decay. This is easily done using masses given in **Appendix A**.

Solution

The decay equation was given earlier for ^{239}Pu; it is

$$^{239}\text{Pu} \rightarrow {}^{235}\text{U} + {}^{4}\text{He}. \tag{31.17}$$

Thus the pertinent masses are those of ^{239}Pu, ^{235}U, and the α particle or ^{4}He, all of which are listed in **Appendix A**. The initial mass was $m(^{239}\text{Pu}) = 239.052157\text{ u}$. The final mass is the sum $m(^{235}\text{U}) + m(^{4}\text{He}) = 235.043924\text{ u} + 4.002602\text{ u} = 239.046526\text{ u}$. Thus,

$$\begin{aligned} \Delta m &= m(^{239}\text{Pu}) - [m(^{235}\text{U}) + m(^{4}\text{He})] \\ &= 239.052157\text{ u} - 239.046526\text{ u} \\ &= 0.0005631\text{ u}. \end{aligned} \tag{31.18}$$

Now we can find E by entering Δm into the equation:

$$E = (\Delta m)c^{2} = (0.005631\text{ u})c^{2}. \tag{31.19}$$

We know $1\text{ u} = 931.5\text{ MeV}/c^{2}$, and so

$$E = (0.005631)(931.5\text{ MeV}/c^2)(c^2) = 5.25\text{ MeV}. \tag{31.20}$$

Discussion

The energy released in this α decay is in the MeV range, about 10^6 times as great as typical chemical reaction energies, consistent with many previous discussions. Most of this energy becomes kinetic energy of the α particle (or ^4He nucleus), which moves away at high speed. The energy carried away by the recoil of the ^{235}U nucleus is much smaller in order to conserve momentum. The ^{235}U nucleus can be left in an excited state to later emit photons (γ rays). This decay is spontaneous and releases energy, because the products have less mass than the parent nucleus. The question of why the products have less mass will be discussed in **Binding Energy**. Note that the masses given in **Appendix A** are atomic masses of neutral atoms, including their electrons. The mass of the electrons is the same before and after α decay, and so their masses subtract out when finding Δm . In this case, there are 94 electrons before and after the decay.

Beta Decay

There are actually *three* types of **beta decay**. The first discovered was "ordinary" beta decay and is called β^- decay or electron emission. The symbol β^- represents *an electron emitted in nuclear beta decay*. Cobalt-60 is a nuclide that β^- decays in the following manner:

$$^{60}\text{Co} \rightarrow {}^{60}\text{Ni} + \beta^- + \text{neutrino}. \tag{31.21}$$

The **neutrino** is a particle emitted in beta decay that was unanticipated and is of fundamental importance. The neutrino was not even proposed in theory until more than 20 years after beta decay was known to involve electron emissions. Neutrinos are so difficult to detect that the first direct evidence of them was not obtained until 1953. Neutrinos are nearly massless, have no charge, and do not interact with nucleons via the strong nuclear force. Traveling approximately at the speed of light, they have little time to affect any nucleus they encounter. This is, owing to the fact that they have no charge (and they are not EM waves), they do not interact through the EM force. They do interact via the relatively weak and very short range weak nuclear force. Consequently, neutrinos escape almost any detector and penetrate almost any shielding. However, neutrinos do carry energy, angular momentum (they are fermions with half-integral spin), and linear momentum away from a beta decay. When accurate measurements of beta decay were made, it became apparent that energy, angular momentum, and linear momentum were not accounted for by the daughter nucleus and electron alone. Either a previously unsuspected particle was carrying them away, or three conservation laws were being violated. Wolfgang Pauli made a formal proposal for the existence of neutrinos in 1930. The Italian-born American physicist Enrico Fermi (1901–1954) gave neutrinos their name, meaning little neutral ones, when he developed a sophisticated theory of beta decay (see **Figure 31.18**). Part of Fermi's theory was the identification of the weak nuclear force as being distinct from the strong nuclear force and in fact responsible for beta decay.

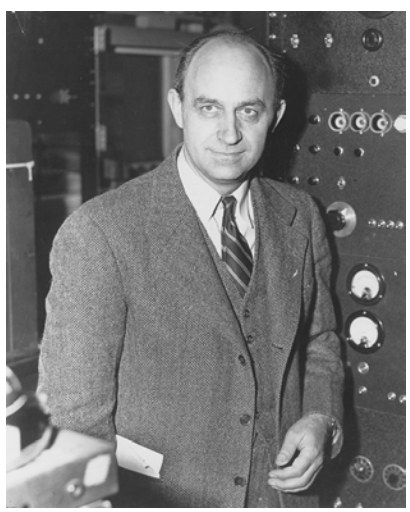


Figure 31.18 Enrico Fermi was nearly unique among 20th-century physicists—he made significant contributions both as an experimentalist and a theorist. His many contributions to theoretical physics included the identification of the weak nuclear force. The fermi (fm) is named after him, as are an entire class of subatomic particles (fermions), an element (Fermium), and a major research laboratory (Fermilab). His experimental work included studies of radioactivity, for which he won the 1938 Nobel Prize in physics, and creation of the first nuclear chain reaction. (credit: United States Department of Energy, Office of Public Affairs)

The neutrino also reveals a new conservation law. There are various families of particles, one of which is the electron family. We propose that the number of members of the electron family is constant in any process or any closed system. In our example of beta decay, there are no members of the electron family present before the decay, but after, there is an electron and a neutrino. So electrons are given an electron family number of $+1$. The neutrino in β^- decay is an **electron's antineutrino**, given the symbol $\bar{\nu}_e$, where ν is the Greek letter nu, and the subscript *e* means this neutrino is related to the electron. The bar indicates this is a particle of **antimatter**. (All particles have antimatter counterparts that are nearly identical except that they have the opposite charge. Antimatter is almost entirely absent on Earth, but it is found in nuclear decay and other nuclear and particle reactions as well as in outer space.) The electron's antineutrino $\bar{\nu}_e$, being antimatter, has an electron family number of -1 . The total is zero, before and after the decay. The new conservation law, obeyed in all circumstances, states that the *total electron family number is constant*. An electron cannot be created without also creating an antimatter family member. This law is analogous to the conservation of charge in a situation where total charge is originally zero, and equal amounts of positive and negative charge must be created in a reaction to keep the total zero.

If a nuclide ${}^{A}_{Z}\mathrm{X}_N$ is known to β^- decay, then its β^- decay equation is

$$\mathrm{X}_N \rightarrow \mathrm{Y}_{N-1} + \beta^- + \bar{\nu}_e \, (\beta^- \text{ decay}), \tag{31.22}$$

where Y is the nuclide having one more proton than X (see **Figure 31.19**). So if you know that a certain nuclide β^- decays, you can find the daughter nucleus by first looking up Z for the parent and then determining which element has atomic number $Z+1$. In the example of the β^- decay of ${}^{60}\mathrm{Co}$ given earlier, we see that $Z = 27$ for Co and $Z = 28$ is Ni. It is as if one of the neutrons in the parent nucleus decays into a proton, electron, and neutrino. In fact, neutrons outside of nuclei do just that—they live only an average of a few minutes and β^- decay in the following manner:

$$\mathrm{n} \rightarrow \mathrm{p} + \beta^- + \bar{\nu}_e. \tag{31.23}$$

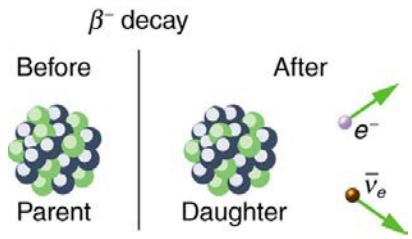


Figure 31.19 In β^- decay, the parent nucleus emits an electron and an antineutrino. The daughter nucleus has one more proton and one less neutron than its parent. Neutrinos interact so weakly that they are almost never directly observed, but they play a fundamental role in particle physics.

We see that charge is conserved in β^- decay, since the total charge is Z before and after the decay. For example, in ${}^{60}\mathrm{Co}$ decay, total charge is 27 before decay, since cobalt has $Z = 27$. After decay, the daughter nucleus is Ni, which has $Z = 28$, and there is an electron, so that the total charge is also $28 + (-1)$ or 27. Angular momentum is conserved, but not obviously (you have to examine the spins and angular momenta of the final products in detail to verify this). Linear momentum is also conserved, again imparting most of the decay energy to the electron and the antineutrino, since they are of low and zero mass, respectively. Another new conservation law is obeyed here and elsewhere in nature. *The total number of nucleons A is conserved*. In ${}^{60}\mathrm{Co}$ decay, for example, there are 60 nucleons before and after the decay. Note that total A is also conserved in α decay. Also note that the total number of protons changes, as does the total number of neutrons, so that total Z and total N are *not* conserved in β^- decay, as they are in α decay. Energy released in β^- decay can be calculated given the masses of the parent and products.

Example 31.3 β^- Decay Energy from Masses

Find the energy emitted in the β^- decay of ${}^{60}\mathrm{Co}$.

Strategy and Concept

As in the preceding example, we must first find Δm, the difference in mass between the parent nucleus and the products of the decay, using masses given in **Appendix A**. Then the emitted energy is calculated as before, using $E = (\Delta m)c^2$. The initial mass is just that of the parent nucleus, and the final mass is that of the daughter nucleus and the electron created in the decay. The neutrino is massless, or nearly so. However, since the masses given in **Appendix A** are for neutral atoms, the daughter nucleus has one more electron than the parent, and so the extra electron mass that corresponds to the β^- is included in the atomic mass of Ni. Thus,

$$\Delta m = m({}^{60}\mathrm{Co}) - m({}^{60}\mathrm{Ni}). \tag{31.24}$$

Solution

The β^- decay equation for ${}^{60}\mathrm{Co}$ is

$${}^{60}_{27}\mathrm{Co}_{33} \rightarrow {}^{60}_{28}\mathrm{Ni}_{32} + \beta^- + \bar{\nu}_e. \tag{31.25}$$

As noticed,

$$\Delta m = m({}^{60}\mathrm{Co}) - m({}^{60}\mathrm{Ni}). \tag{31.26}$$

Entering the masses found in **Appendix A** gives

$$\Delta m = 59.933820 \text{ u} - 59.930789 \text{ u} = 0.003031 \text{ u}. \tag{31.27}$$

Thus,

$$E = (\Delta m)c^2 = (0.003031 \text{ u})c^2. \tag{31.28}$$

Using $1 \text{ u} = 931.5 \text{ MeV}/c^2$, we obtain

$$E = (0.003031)(931.5\text{ MeV}/c^2)(c^2) = 2.82\text{ MeV}. \tag{31.29}$$

Discussion and Implications

Perhaps the most difficult thing about this example is convincing yourself that the β^- mass is included in the atomic mass of ^{60}Ni. Beyond that are other implications. Again the decay energy is in the MeV range. This energy is shared by all of the products of the decay. In many ^{60}Co decays, the daughter nucleus ^{60}Ni is left in an excited state and emits photons (γ rays). Most of the remaining energy goes to the electron and neutrino, since the recoil kinetic energy of the daughter nucleus is small. One final note: the electron emitted in β^- decay is created in the nucleus at the time of decay.

The second type of beta decay is less common than the first. It is β^+ decay. Certain nuclides decay by the emission of a *positive* electron. This is **antielectron** or **positron decay** (see **Figure 31.20**).

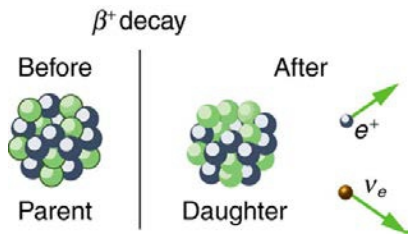


Figure 31.20 β^+ decay is the emission of a positron that eventually finds an electron to annihilate, characteristically producing gammas in opposite directions.

The antielectron is often represented by the symbol e^+, but in beta decay it is written as β^+ to indicate the antielectron was emitted in a nuclear decay. Antielectrons are the antimatter counterpart to electrons, being nearly identical, having the same mass, spin, and so on, but having a positive charge and an electron family number of -1. When a **positron** encounters an electron, there is a mutual annihilation in which all the mass of the antielectron-electron pair is converted into pure photon energy. (The reaction, $e^+ + e^- \rightarrow \gamma + \gamma$, conserves electron family number as well as all other conserved quantities.) If a nuclide $^A_Z\text{X}_N$ is known to β^+ decay, then its β^+ **decay equation** is

$$^A_Z\text{X}_N \rightarrow \text{Y}_{N+1} + \beta^+ + \nu_e\ (\beta^+\text{ decay}), \tag{31.30}$$

where Y is the nuclide having one less proton than X (to conserve charge) and ν_e is the symbol for the **electron's neutrino**, which has an electron family number of $+1$. Since an antimatter member of the electron family (the β^+) is created in the decay, a matter member of the family (here the ν_e) must also be created. Given, for example, that ^{22}Na β^+ decays, you can write its full decay equation by first finding that $Z = 11$ for ^{22}Na, so that the daughter nuclide will have $Z = 10$, the atomic number for neon. Thus the β^+ decay equation for ^{22}Na is

$$^{22}_{11}\text{Na}_{11} \rightarrow {}^{22}_{10}\text{Ne}_{12} + \beta^+ + \nu_e. \tag{31.31}$$

In β^+ decay, it is as if one of the protons in the parent nucleus decays into a neutron, a positron, and a neutrino. Protons do not do this outside of the nucleus, and so the decay is due to the complexities of the nuclear force. Note again that the total number of nucleons is constant in this and any other reaction. To find the energy emitted in β^+ decay, you must again count the number of electrons in the neutral atoms, since atomic masses are used. The daughter has one less electron than the parent, and one electron mass is created in the decay. Thus, in β^+ decay,

$$\Delta m = m(\text{parent}) - [m(\text{daughter}) + 2m_e], \tag{31.32}$$

since we use the masses of neutral atoms.

Electron capture is the third type of beta decay. Here, a nucleus captures an inner-shell electron and undergoes a nuclear reaction that has the same effect as β^+ decay. Electron capture is sometimes denoted by the letters EC. We know that electrons cannot reside in the nucleus, but this is a nuclear reaction that consumes the electron and occurs spontaneously only when the products have less mass than the parent plus the electron. If a nuclide $^A_Z\text{X}_N$ is known to undergo electron capture, then its **electron capture equation** is

$$^A_Z\text{X}_N + e^- \rightarrow \text{Y}_{N+1} + \nu_e(\text{electron capture, or EC}). \tag{31.33}$$

Any nuclide that can β^+ decay can also undergo electron capture (and often does both). The same conservation laws are obeyed for EC as for β^+ decay. It is good practice to confirm these for yourself.

All forms of beta decay occur because the parent nuclide is unstable and lies outside the region of stability in the chart of nuclides. Those nuclides that have relatively more neutrons than those in the region of stability will β^- decay to produce a daughter with fewer neutrons, producing a

daughter nearer the region of stability. Similarly, those nuclides having relatively more protons than those in the region of stability will β^- decay or undergo electron capture to produce a daughter with fewer protons, nearer the region of stability.

Gamma Decay

Gamma decay is the simplest form of nuclear decay—it is the emission of energetic photons by nuclei left in an excited state by some earlier process. Protons and neutrons in an excited nucleus are in higher orbitals, and they fall to lower levels by photon emission (analogous to electrons in excited atoms). Nuclear excited states have lifetimes typically of only about 10^{-14} s, an indication of the great strength of the forces pulling the nucleons to lower states. The γ decay equation is simply

$$ {}^{A}_{Z}\mathrm{X}^{*}_{N} \rightarrow \mathrm{X}_{N} + \gamma_1 + \gamma_2 + \cdots \ (\gamma \text{ decay}) \tag{31.34} $$

where the asterisk indicates the nucleus is in an excited state. There may be one or more γ s emitted, depending on how the nuclide de-excites. In radioactive decay, γ emission is common and is preceded by γ or β decay. For example, when ${}^{60}\mathrm{Co}$ β^- decays, it most often leaves the daughter nucleus in an excited state, written ${}^{60}\mathrm{Ni}^*$. Then the nickel nucleus quickly γ decays by the emission of two penetrating γ s:

$$ {}^{60}\mathrm{Ni}^* \rightarrow {}^{60}\mathrm{Ni} + \gamma_1 + \gamma_2. \tag{31.35} $$

These are called cobalt γ rays, although they come from nickel—they are used for cancer therapy, for example. It is again constructive to verify the conservation laws for gamma decay. Finally, since γ decay does not change the nuclide to another species, it is not prominently featured in charts of decay series, such as that in **Figure 31.16**.

There are other types of nuclear decay, but they occur less commonly than α, β, and γ decay. Spontaneous fission is the most important of the other forms of nuclear decay because of its applications in nuclear power and weapons. It is covered in the next chapter.

31.5 Half-Life and Activity

Unstable nuclei decay. However, some nuclides decay faster than others. For example, radium and polonium, discovered by the Curies, decay faster than uranium. This means they have shorter lifetimes, producing a greater rate of decay. In this section we explore half-life and activity, the quantitative terms for lifetime and rate of decay.

Half-Life

Why use a term like half-life rather than lifetime? The answer can be found by examining **Figure 31.21**, which shows how the number of radioactive nuclei in a sample decreases with time. The *time in which half of the original number of nuclei decay* is defined as the **half-life**, $t_{1/2}$. Half of the remaining nuclei decay in the next half-life. Further, half of that amount decays in the following half-life. Therefore, the number of radioactive nuclei decreases from N to $N/2$ in one half-life, then to $N/4$ in the next, and to $N/8$ in the next, and so on. If N is a large number, then *many* half-lives (not just two) pass before all of the nuclei decay. Nuclear decay is an example of a purely statistical process. A more precise definition of half-life is that *each nucleus has a 50% chance of living for a time equal to one half-life* $t_{1/2}$. Thus, if N is reasonably large, half of the original nuclei decay in a time of one half-life. If an individual nucleus makes it through that time, it still has a 50% chance of surviving through another half-life. Even if it happens to make it through hundreds of half-lives, it still has a 50% chance of surviving through one more. The probability of decay is the same no matter when you start counting. This is like random coin flipping. The chance of heads is 50%, no matter what has happened before.

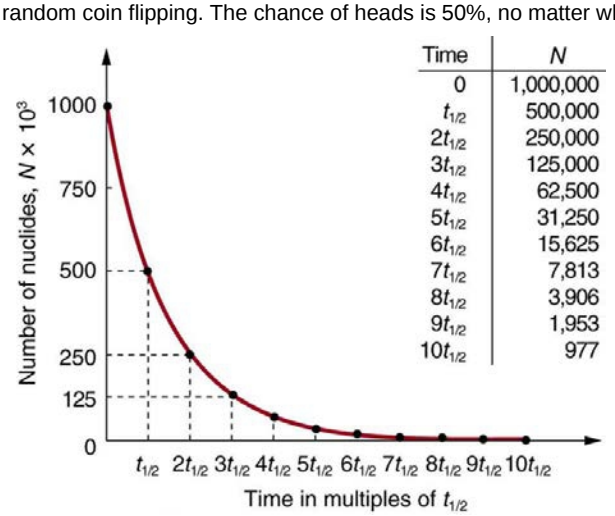


Figure 31.21 Radioactive decay reduces the number of radioactive nuclei over time. In one half-life $t_{1/2}$, the number decreases to half of its original value. Half of what remains decay in the next half-life, and half of those in the next, and so on. This is an exponential decay, as seen in the graph of the number of nuclei present as a function of time.

There is a tremendous range in the half-lives of various nuclides, from as short as 10^{-23} s for the most unstable, to more than 10^{16} y for the least unstable, or about 46 orders of magnitude. Nuclides with the shortest half-lives are those for which the nuclear forces are least attractive, an

indication of the extent to which the nuclear force can depend on the particular combination of neutrons and protons. The concept of half-life is applicable to other subatomic particles, as will be discussed in **Particle Physics**. It is also applicable to the decay of excited states in atoms and nuclei. The following equation gives the quantitative relationship between the original number of nuclei present at time zero (N_0) and the number (N) at a later time t :

$$N = N_0 e^{-\lambda t}, \tag{31.36}$$

where $e = 2.71828...$ is the base of the natural logarithm, and λ is the **decay constant** for the nuclide. The shorter the half-life, the larger is the value of λ , and the faster the exponential $e^{-\lambda t}$ decreases with time. The relationship between the decay constant λ and the half-life $t_{1/2}$ is

$$\lambda = \frac{\ln(2)}{t_{1/2}} \approx \frac{0.693}{t_{1/2}}. \tag{31.37}$$

To see how the number of nuclei declines to half its original value in one half-life, let $t = t_{1/2}$ in the exponential in the equation $N = N_0 e^{-\lambda t}$. This gives $N = N_0 e^{-\lambda t} = N_0 e^{-0.693} = 0.500 N_0$. For integral numbers of half-lives, you can just divide the original number by 2 over and over again, rather than using the exponential relationship. For example, if ten half-lives have passed, we divide N by 2 ten times. This reduces it to $N/1024$. For an arbitrary time, not just a multiple of the half-life, the exponential relationship must be used.

Radioactive dating is a clever use of naturally occurring radioactivity. Its most famous application is **carbon-14 dating**. Carbon-14 has a half-life of 5730 years and is produced in a nuclear reaction induced when solar neutrinos strike ^{14}N in the atmosphere. Radioactive carbon has the same chemistry as stable carbon, and so it mixes into the ecosphere, where it is consumed and becomes part of every living organism. Carbon-14 has an abundance of 1.3 parts per trillion of normal carbon. Thus, if you know the number of carbon nuclei in an object (perhaps determined by mass and Avogadro's number), you multiply that number by 1.3×10^{-12} to find the number of ^{14}C nuclei in the object. When an organism dies, carbon exchange with the environment ceases, and ^{14}C is not replenished as it decays. By comparing the abundance of ^{14}C in an artifact, such as mummy wrappings, with the normal abundance in living tissue, it is possible to determine the artifact's age (or time since death). Carbon-14 dating can be used for biological tissues as old as 50 or 60 thousand years, but is most accurate for younger samples, since the abundance of ^{14}C nuclei in them is greater. Very old biological materials contain no ^{14}C at all. There are instances in which the date of an artifact can be determined by other means, such as historical knowledge or tree-ring counting. These cross-references have confirmed the validity of carbon-14 dating and permitted us to calibrate the technique as well. Carbon-14 dating revolutionized parts of archaeology and is of such importance that it earned the 1960 Nobel Prize in chemistry for its developer, the American chemist Willard Libby (1908–1980).

One of the most famous cases of carbon-14 dating involves the Shroud of Turin, a long piece of fabric purported to be the burial shroud of Jesus (see **Figure 31.22**). This relic was first displayed in Turin in 1354 and was denounced as a fraud at that time by a French bishop. Its remarkable negative imprint of an apparently crucified body resembles the then-accepted image of Jesus, and so the shroud was never disregarded completely and remained controversial over the centuries. Carbon-14 dating was not performed on the shroud until 1988, when the process had been refined to the point where only a small amount of material needed to be destroyed. Samples were tested at three independent laboratories, each being given four pieces of cloth, with only one unidentified piece from the shroud, to avoid prejudice. All three laboratories found samples of the shroud contain 92% of the ^{14}C found in living tissues, allowing the shroud to be dated (see **Example 31.4**).

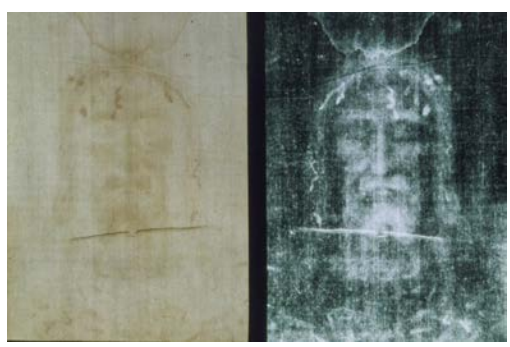


Figure 31.22 Part of the Shroud of Turin, which shows a remarkable negative imprint likeness of Jesus complete with evidence of crucifixion wounds. The shroud first surfaced in the 14th century and was only recently carbon-14 dated. It has not been determined how the image was placed on the material. (credit: Butko, Wikimedia Commons)

Example 31.4 How Old Is the Shroud of Turin?

Calculate the age of the Shroud of Turin given that the amount of ^{14}C found in it is 92% of that in living tissue.

Strategy

Knowing that 92% of the ^{14}C remains means that $N/N_0 = 0.92$. Therefore, the equation $N = N_0 e^{-\lambda t}$ can be used to find λt. We also know that the half-life of ^{14}C is 5730 y, and so once λt is known, we can use the equation $\lambda = \frac{0.693}{t_{1/2}}$ to find λ and then find t as requested. Here, we postulate that the decrease in ^{14}C is solely due to nuclear decay.

Solution

Solving the equation $N = N_0 e^{-\lambda t}$ for N/N_0 gives

$$\frac{N}{N_0} = e^{-\lambda t}. \tag{31.38}$$

Thus,

$$0.92 = e^{-\lambda t}. \tag{31.39}$$

Taking the natural logarithm of both sides of the equation yields

$$\ln 0.92 = -\lambda t \tag{31.40}$$

so that

$$-0.0834 = -\lambda t. \tag{31.41}$$

Rearranging to isolate t gives

$$t = \frac{0.0834}{\lambda}. \tag{31.42}$$

Now, the equation $\lambda = \frac{0.693}{t_{1/2}}$ can be used to find λ for ^{14}C. Solving for λ and substituting the known half-life gives

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{5730\text{ y}}. \tag{31.43}$$

We enter this value into the previous equation to find t:

$$t = \frac{0.0834}{\frac{0.693}{5730\text{ y}}} = 690\text{ y}. \tag{31.44}$$

Discussion

This dates the material in the shroud to 1988–690 = a.d. 1300. Our calculation is only accurate to two digits, so that the year is rounded to 1300. The values obtained at the three independent laboratories gave a weighted average date of a.d. 1320 ± 60. The uncertainty is typical of carbon-14 dating and is due to the small amount of ^{14}C in living tissues, the amount of material available, and experimental uncertainties (reduced by having three independent measurements). It is meaningful that the date of the shroud is consistent with the first record of its existence and inconsistent with the period in which Jesus lived.

There are other forms of radioactive dating. Rocks, for example, can sometimes be dated based on the decay of ^{238}U. The decay series for ^{238}U ends with ^{206}Pb, so that the ratio of these nuclides in a rock is an indication of how long it has been since the rock solidified. The original composition of the rock, such as the absence of lead, must be known with some confidence. However, as with carbon-14 dating, the technique can be verified by a consistent body of knowledge. Since ^{238}U has a half-life of 4.5×10^9 y, it is useful for dating only very old materials, showing, for example, that the oldest rocks on Earth solidified about 3.5×10^9 years ago.

Activity, the Rate of Decay

What do we mean when we say a source is highly radioactive? Generally, this means the number of decays per unit time is very high. We define **activity** R to be the **rate of decay** expressed in decays per unit time. In equation form, this is

$$R = \frac{\Delta N}{\Delta t} \tag{31.45}$$

where ΔN is the number of decays that occur in time Δt. The SI unit for activity is one decay per second and is given the name **becquerel** (Bq) in honor of the discoverer of radioactivity. That is,

$$1\text{ Bq} = 1\text{ decay/s}. \tag{31.46}$$

Activity R is often expressed in other units, such as decays per minute or decays per year. One of the most common units for activity is the **curie** (Ci), defined to be the activity of 1 g of ^{226}Ra, in honor of Marie Curie's work with radium. The definition of curie is

$$1\text{ Ci} = 3.70\times10^{10}\text{ Bq}, \tag{31.47}$$

or 3.70×10^{10} decays per second. A curie is a large unit of activity, while a becquerel is a relatively small unit. $1\text{ MBq} = 100\text{ microcuries }(\mu\text{Ci})$. In countries like Australia and New Zealand that adhere more to SI units, most radioactive sources, such as those used in medical diagnostics or in physics laboratories, are labeled in Bq or megabecquerel (MBq).

Intuitively, you would expect the activity of a source to depend on two things: the amount of the radioactive substance present, and its half-life. The greater the number of radioactive nuclei present in the sample, the more will decay per unit of time. The shorter the half-life, the more decays per unit time, for a given number of nuclei. So activity R should be proportional to the number of radioactive nuclei, N, and inversely proportional to their half-life, $t_{1/2}$. In fact, your intuition is correct. It can be shown that the activity of a source is

$$R = \frac{0.693N}{t_{1/2}} \tag{31.48}$$

where N is the number of radioactive nuclei present, having half-life $t_{1/2}$. This relationship is useful in a variety of calculations, as the next two examples illustrate.

Example 31.5 How Great Is the ^{14}C Activity in Living Tissue?

Calculate the activity due to ^{14}C in 1.00 kg of carbon found in a living organism. Express the activity in units of Bq and Ci.

Strategy

To find the activity R using the equation $R = \frac{0.693N}{t_{1/2}}$, we must know N and $t_{1/2}$. The half-life of ^{14}C can be found in **Appendix B**, and was stated above as 5730 y. To find N, we first find the number of ^{12}C nuclei in 1.00 kg of carbon using the concept of a mole. As indicated, we then multiply by 1.3×10^{-12} (the abundance of ^{14}C in a carbon sample from a living organism) to get the number of ^{14}C nuclei in a living organism.

Solution

One mole of carbon has a mass of 12.0 g, since it is nearly pure ^{12}C. (A mole has a mass in grams equal in magnitude to A found in the periodic table.) Thus the number of carbon nuclei in a kilogram is

$$N(^{12}\text{C}) = \frac{6.02\times10^{23}\text{ mol}^{-1}}{12.0\text{ g/mol}}\times(1000\text{ g}) = 5.02\times10^{25}. \tag{31.49}$$

So the number of ^{14}C nuclei in 1 kg of carbon is

$$N(^{14}\text{C}) = (5.02\times10^{25})(1.3\times10^{-12}) = 6.52\times10^{13}. \tag{31.50}$$

Now the activity R is found using the equation $R = \frac{0.693N}{t_{1/2}}$.

Entering known values gives

$$R = \frac{0.693(6.52\times10^{13})}{5730\text{ y}} = 7.89\times10^{9}\text{ y}^{-1}, \tag{31.51}$$

or 7.89×10^{9} decays per year. To convert this to the unit Bq, we simply convert years to seconds. Thus,

$$R = (7.89\times10^{9}\text{ y}^{-1})\frac{1.00\text{ y}}{3.16\times10^{7}\text{ s}} = 250\text{ Bq}, \tag{31.52}$$

or 250 decays per second. To express R in curies, we use the definition of a curie,

$$R = \frac{250\text{ Bq}}{3.7\times10^{10}\text{ Bq/Ci}} = 6.76\times10^{-9}\text{ Ci}. \tag{31.53}$$

Thus,

$$R = 6.76\text{ nCi}. \tag{31.54}$$

Discussion

Our own bodies contain kilograms of carbon, and it is intriguing to think there are hundreds of ^{14}C decays per second taking place in us. Carbon-14 and other naturally occurring radioactive substances in our bodies contribute to the background radiation we receive. The small number of decays per second found for a kilogram of carbon in this example gives you some idea of how difficult it is to detect ^{14}C in a small sample of material. If there are 250 decays per second in a kilogram, then there are 0.25 decays per second in a gram of carbon in living tissue.

To observe this, you must be able to distinguish decays from other forms of radiation, in order to reduce background noise. This becomes more difficult with an old tissue sample, since it contains less ^{14}C, and for samples more than 50 thousand years old, it is impossible.

Human-made (or artificial) radioactivity has been produced for decades and has many uses. Some of these include medical therapy for cancer, medical imaging and diagnostics, and food preservation by irradiation. Many applications as well as the biological effects of radiation are explored in **Medical Applications of Nuclear Physics**, but it is clear that radiation is hazardous. A number of tragic examples of this exist, one of the most disastrous being the meltdown and fire at the Chernobyl reactor complex in the Ukraine (see **Figure 31.23**). Several radioactive isotopes were released in huge quantities, contaminating many thousands of square kilometers and directly affecting hundreds of thousands of people. The most significant releases were of ^{131}I, ^{90}Sr, ^{137}Cs, ^{239}Pu, ^{238}U, and ^{235}U. Estimates are that the total amount of radiation released was about 100 million curies.

Human and Medical Applications

Figure 31.23 The Chernobyl reactor. More than 100 people died soon after its meltdown, and there will be thousands of deaths from radiation-induced cancer in the future. While the accident was due to a series of human errors, the cleanup efforts were heroic. Most of the immediate fatalities were firefighters and reactor personnel. (credit: Elena Filatova)

Example 31.6 What Mass of ^{137}Cs Escaped Chernobyl?

It is estimated that the Chernobyl disaster released 6.0 MCi of ^{137}Cs into the environment. Calculate the mass of ^{137}Cs released.

Strategy

We can calculate the mass released using Avogadro's number and the concept of a mole if we can first find the number of nuclei N released. Since the activity R is given, and the half-life of ^{137}Cs is found in **Appendix B** to be 30.2 y, we can use the equation $R = \frac{0.693N}{t_{1/2}}$ to find N.

Solution

Solving the equation $R = \frac{0.693N}{t_{1/2}}$ for N gives

$$N = \frac{Rt_{1/2}}{0.693}. \tag{31.55}$$

Entering the given values yields

$$N = \frac{(6.0\text{ MCi})(30.2\text{ y})}{0.693}. \tag{31.56}$$

Converting curies to becquerels and years to seconds, we get

$$\begin{aligned} N &= \frac{(6.0\times10^{6}\text{ Ci})(3.7\times10^{10}\text{ Bq/Ci})(30.2\text{ y})(3.16\times10^{7}\text{ s/y})}{0.693} \\ &= 3.1\times10^{26}. \end{aligned} \tag{31.57}$$

One mole of a nuclide ^{A}X has a mass of A grams, so that one mole of ^{137}Cs has a mass of 137 g. A mole has 6.02×10^{23} nuclei. Thus the mass of ^{137}Cs released was

$$\begin{aligned} m &= \left(\frac{137\text{ g}}{6.02\times10^{23}}\right)(3.1\times10^{26}) = 70\times10^{3}\text{ g} \\ &= 70\text{ kg}. \end{aligned} \tag{31.58}$$

Discussion

While 70 kg of material may not be a very large mass compared to the amount of fuel in a power plant, it is extremely radioactive, since it only has a 30-year half-life. Six megacuries (6.0 MCi) is an extraordinary amount of activity but is only a fraction of what is produced in nuclear reactors. Similar amounts of the other isotopes were also released at Chernobyl. Although the chances of such a disaster may have seemed small, the consequences were extremely severe, requiring greater caution than was used. More will be said about safe reactor design in the next chapter, but it should be noted that Western reactors have a fundamentally safer design.

Activity R decreases in time, going to half its original value in one half-life, then to one-fourth its original value in the next half-life, and so on. Since $R = \frac{0.693N}{t_{1/2}}$, the activity decreases as the number of radioactive nuclei decreases. The equation for R as a function of time is found by combining the equations $N = N_0 e^{-\lambda t}$ and $R = \frac{0.693N}{t_{1/2}}$, yielding

$$R = R_0 e^{-\lambda t}, \tag{31.59}$$

where R_0 is the activity at $t = 0$. This equation shows exponential decay of radioactive nuclei. For example, if a source originally has a 1.00-mCi activity, it declines to 0.500 mCi in one half-life, to 0.250 mCi in two half-lives, to 0.125 mCi in three half-lives, and so on. For times other than whole half-lives, the equation $R = R_0 e^{-\lambda t}$ must be used to find R.

PhET Explorations: Alpha Decay

Watch alpha particles escape from a polonium nucleus, causing radioactive alpha decay. See how random decay times relate to the half life.

PhET Interactive Simulation

Figure 31.24 Alpha Decay (http://cnx.org/content/m42636/1.6/alpha-decay_en.jar)

31.6 Binding Energy

The more tightly bound a system is, the stronger the forces that hold it together and the greater the energy required to pull it apart. We can therefore learn about nuclear forces by examining how tightly bound the nuclei are. We define the **binding energy** (BE) of a nucleus to be *the energy required to completely disassemble it into separate protons and neutrons*. We can determine the BE of a nucleus from its rest mass. The two are connected through Einstein's famous relationship $E = (\Delta m)c^2$. A bound system has a *smaller* mass than its separate constituents; the more tightly the nucleons are bound together, the smaller the mass of the nucleus.

Imagine pulling a nuclide apart as illustrated in **Figure 31.25**. Work done to overcome the nuclear forces holding the nucleus together puts energy into the system. By definition, the energy input equals the binding energy BE. The pieces are at rest when separated, and so the energy put into them increases their total rest mass compared with what it was when they were glued together as a nucleus. That mass increase is thus $\Delta m = \text{BE}/c^2$. This difference in mass is known as *mass defect*. It implies that the mass of the nucleus is less than the sum of the masses of its constituent protons and neutrons. A nuclide ${}^A\text{X}$ has Z protons and N neutrons, so that the difference in mass is

$$\Delta m = (Zm_p + Nm_n) - m_{\text{tot}}. \tag{31.60}$$

Thus,

$$\text{BE} = (\Delta m)c^2 = [(Zm_p + Nm_n) - m_{\text{tot}}]c^2, \tag{31.61}$$

where m_{tot} is the mass of the nuclide ${}^A\text{X}$, m_p is the mass of a proton, and m_n is the mass of a neutron. Traditionally, we deal with the masses of neutral atoms. To get atomic masses into the last equation, we first add Z electrons to m_{tot}, which gives $m\left({}^A\text{X}\right)$, the atomic mass of the nuclide. We then add Z electrons to the Z protons, which gives $Zm\left({}^1\text{H}\right)$, or Z times the mass of a hydrogen atom. Thus the binding energy of a nuclide ${}^A\text{X}$ is

$$\text{BE} = \left\{[Zm({}^1\text{H}) + Nm_n] - m({}^A X)\right\}c^2. \tag{31.62}$$

The atomic masses can be found in **Appendix A**, most conveniently expressed in unified atomic mass units u ($1\text{ u} = 931.5\text{ MeV}/c^2$). BE is thus calculated from known atomic masses.

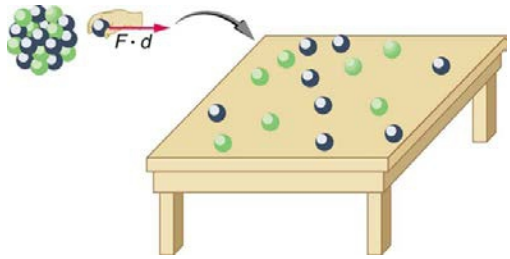


Figure 31.25 Work done to pull a nucleus apart into its constituent protons and neutrons increases the mass of the system. The work to disassemble the nucleus equals its binding energy BE. A bound system has less mass than the sum of its parts, especially noticeable in the nuclei, where forces and energies are very large.

Things Great and Small

Nuclear Decay Helps Explain Earth's Hot Interior

A puzzle created by radioactive dating of rocks is resolved by radioactive heating of Earth's interior. This intriguing story is another example of how small-scale physics can explain large-scale phenomena.

Radioactive dating plays a role in determining the approximate age of the Earth. The oldest rocks on Earth solidified about 3.5×10^9 years ago—a number determined by uranium-238 dating. These rocks could only have solidified once the surface of the Earth had cooled sufficiently. The temperature of the Earth at formation can be estimated based on gravitational potential energy of the assemblage of pieces being converted to thermal energy. Using heat transfer concepts discussed in **Thermodynamics** it is then possible to calculate how long it would take for the surface to cool to rock-formation temperatures. The result is about 10^9 years. The first rocks formed have been solid for 3.5×10^9 years, so that the age of the Earth is approximately 4.5×10^9 years. There is a large body of other types of evidence (both Earth-bound and solar system characteristics are used) that supports this age. The puzzle is that, given its age and initial temperature, the center of the Earth should be much cooler than it is today (see **Figure 31.26**).

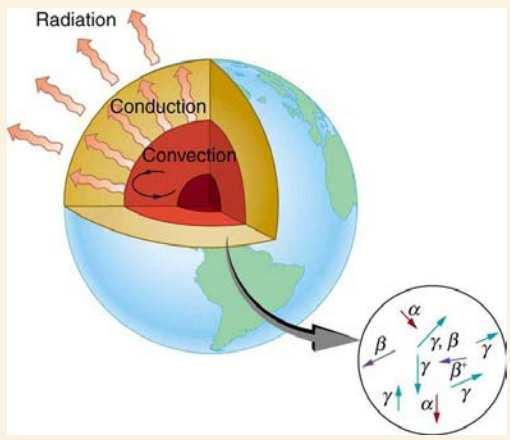


Figure 31.26 The center of the Earth cools by well-known heat transfer methods. Convection in the liquid regions and conduction move thermal energy to the surface, where it radiates into cold, dark space. Given the age of the Earth and its initial temperature, it should have cooled to a lower temperature by now. The blowup shows that nuclear decay releases energy in the Earth's interior. This energy has slowed the cooling process and is responsible for the interior still being molten.

We know from seismic waves produced by earthquakes that parts of the interior of the Earth are liquid. Shear or transverse waves cannot travel through a liquid and are not transmitted through the Earth's core. Yet compression or longitudinal waves can pass through a liquid and do go through the core. From this information, the temperature of the interior can be estimated. As noticed, the interior should have cooled more from its initial temperature in the 4.5×10^9 years since its formation. In fact, it should have taken no more than about 10^9 years to cool to its present temperature. What is keeping it hot? The answer seems to be radioactive decay of primordial elements that were part of the material that formed the Earth (see the blowup in **Figure 31.26**).

Nuclides such as ^{238}U and ^{40}K have half-lives similar to or longer than the age of the Earth, and their decay still contributes energy to the interior. Some of the primordial radioactive nuclides have unstable decay products that also release energy— ^{238}U has a long decay chain of these. Further, there were more of these primordial radioactive nuclides early in the life of the Earth, and thus the activity and energy contributed were greater then (perhaps by an order of magnitude). The amount of power created by these decays per cubic meter is very small. However, since a huge volume of material lies deep below the surface, this relatively small amount of energy cannot escape quickly. The power produced near the surface has much less distance to go to escape and has a negligible effect on surface temperatures.

A final effect of this trapped radiation merits mention. Alpha decay produces helium nuclei, which form helium atoms when they are stopped and capture electrons. Most of the helium on Earth is obtained from wells and is produced in this manner. Any helium in the atmosphere will escape in geologically short times because of its high thermal velocity.

What patterns and insights are gained from an examination of the binding energy of various nuclides? First, we find that BE is approximately proportional to the number of nucleons A in any nucleus. About twice as much energy is needed to pull apart a nucleus like ^{24}Mg compared with pulling apart ^{12}C, for example. To help us look at other effects, we divide BE by A and consider the **binding energy per nucleon**, BE/A. The

graph of BE/A in **Figure 31.27** reveals some very interesting aspects of nuclei. We see that the binding energy per nucleon averages about 8 MeV, but is lower for both the lightest and heaviest nuclei. This overall trend, in which nuclei with A equal to about 60 have the greatest BE/A and are thus the most tightly bound, is due to the combined characteristics of the attractive nuclear forces and the repulsive Coulomb force. It is especially important to note two things—the strong nuclear force is about 100 times stronger than the Coulomb force, *and* the nuclear forces are shorter in range compared to the Coulomb force. So, for low-mass nuclei, the nuclear attraction dominates and each added nucleon forms bonds with all others, causing progressively heavier nuclei to have progressively greater values of BE/A. This continues up to $A \approx 60$, roughly corresponding to the mass number of iron. Beyond that, new nucleons added to a nucleus will be too far from some others to feel their nuclear attraction. Added protons, however, feel the repulsion of all other protons, since the Coulomb force is longer in range. Coulomb repulsion grows for progressively heavier nuclei, but nuclear attraction remains about the same, and so BE/A becomes smaller. This is why stable nuclei heavier than $A \approx 40$ have more neutrons than protons. Coulomb repulsion is reduced by having more neutrons to keep the protons farther apart (see **Figure 31.28**).

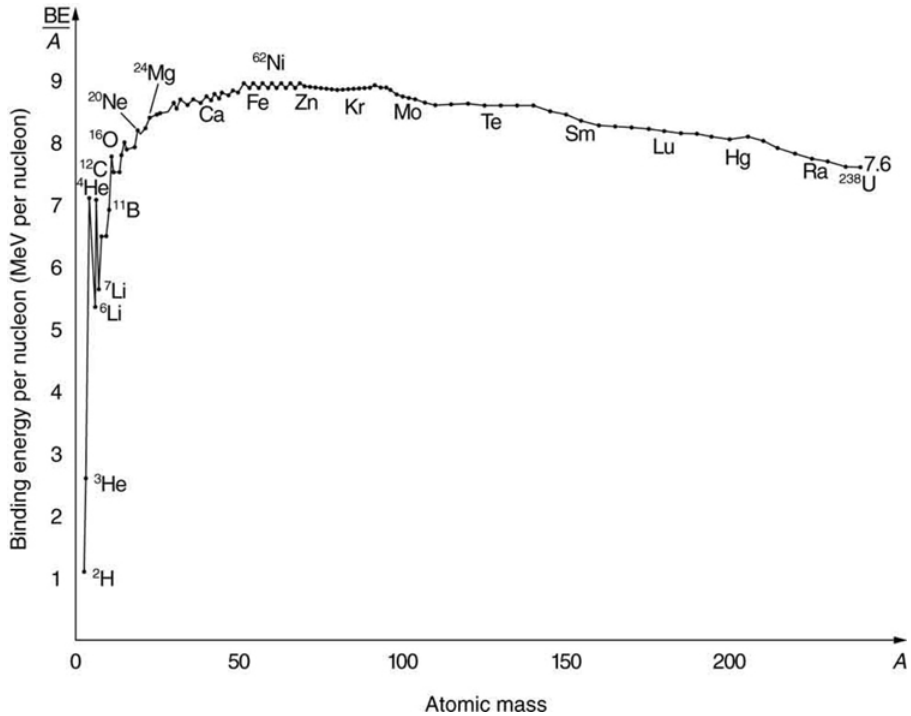


Figure 31.27 A graph of average binding energy per nucleon, BE/A, for stable nuclei. The most tightly bound nuclei are those with A near 60, where the attractive nuclear force has its greatest effect. At higher A s, the Coulomb repulsion progressively reduces the binding energy per nucleon, because the nuclear force is short ranged. The spikes on the curve are very tightly bound nuclides and indicate shell closures.

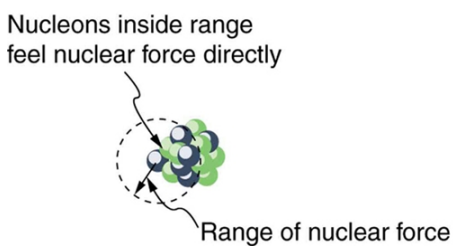


Figure 31.28 The nuclear force is attractive and stronger than the Coulomb force, but it is short ranged. In low-mass nuclei, each nucleon feels the nuclear attraction of all others. In larger nuclei, the range of the nuclear force, shown for a single nucleon, is smaller than the size of the nucleus, but the Coulomb repulsion from all protons reaches all others. If the nucleus is large enough, the Coulomb repulsion can add to overcome the nuclear attraction.

There are some noticeable spikes on the BE/A graph, which represent particularly tightly bound nuclei. These spikes reveal further details of nuclear forces, such as confirming that closed-shell nuclei (those with magic numbers of protons or neutrons or both) are more tightly bound. The spikes also indicate that some nuclei with even numbers for Z and N, and with $Z = N$, are exceptionally tightly bound. This finding can be correlated with some of the cosmic abundances of the elements. The most common elements in the universe, as determined by observations of atomic spectra from outer space, are hydrogen, followed by $^4\mathrm{He}$, with much smaller amounts of $^{12}\mathrm{C}$ and other elements. It should be noted that the heavier elements are created in supernova explosions, while the lighter ones are produced by nuclear fusion during the normal life cycles of stars, as will be discussed in subsequent chapters. The most common elements have the most tightly bound nuclei. It is also no accident that one of the most tightly bound light nuclei is $^4\mathrm{He}$, emitted in α decay.

Example 31.7 What Is BE/A for an Alpha Particle?

Calculate the binding energy per nucleon of $^4\mathrm{He}$, the α particle.

Strategy

To find BE/A, we first find BE using the Equation $\mathrm{BE} = \{[Zm(^1\mathrm{H}) + Nm_n] - m(^A\mathrm{X})\}c^2$ and then divide by A. This is straightforward once we have looked up the appropriate atomic masses in **Appendix A**.

Solution

The binding energy for a nucleus is given by the equation

$$\mathrm{BE} = \{[Zm(^1\mathrm{H}) + Nm_n] - m(^A\mathrm{X})\}c^2. \tag{31.63}$$

For $^4\mathrm{He}$, we have $Z = N = 2$; thus,

$$\mathrm{BE} = \{[2m(^1\mathrm{H}) + 2m_n] - m(^4\mathrm{He})\}c^2. \tag{31.64}$$

Appendix A gives these masses as $m(^4\mathrm{He}) = 4.002602\ \mathrm{u}$, $m(^1\mathrm{H}) = 1.007825\ \mathrm{u}$, and $m_n = 1.008665\ \mathrm{u}$. Thus,

$$\mathrm{BE} = (0.030378\ \mathrm{u})c^2. \tag{31.65}$$

Noting that $1\ \mathrm{u} = 931.5\ \mathrm{MeV}/c^2$, we find

$$\mathrm{BE} = (0.030378)(931.5\ \mathrm{MeV}/c^2)c^2 = 28.3\ \mathrm{MeV}. \tag{31.66}$$

Since $A = 4$, we see that BE/A is this number divided by 4, or

$$\mathrm{BE}/A = 7.07\ \mathrm{MeV/nucleon}. \tag{31.67}$$

Discussion

This is a large binding energy per nucleon compared with those for other low-mass nuclei, which have $\mathrm{BE}/A \approx 3\ \mathrm{MeV/nucleon}$. This indicates that $^4\mathrm{He}$ is tightly bound compared with its neighbors on the chart of the nuclides. You can see the spike representing this value of BE/A for $^4\mathrm{He}$ on the graph in **Figure 31.27**. This is why $^4\mathrm{He}$ is stable. Since $^4\mathrm{He}$ is tightly bound, it has less mass than other $A = 4$ nuclei and, therefore, cannot spontaneously decay into them. The large binding energy also helps to explain why some nuclei undergo α decay. Smaller mass in the decay products can mean energy release, and such decays can be spontaneous. Further, it can happen that two protons and two neutrons in a nucleus can randomly find themselves together, experience the exceptionally large nuclear force that binds this combination, and act as a $^4\mathrm{He}$ unit within the nucleus, at least for a while. In some cases, the $^4\mathrm{He}$ escapes, and α decay has then taken place.

There is more to be learned from nuclear binding energies. The general trend in BE/A is fundamental to energy production in stars, and to fusion and fission energy sources on Earth, for example. This is one of the applications of nuclear physics covered in **Medical Applications of Nuclear Physics**. The abundance of elements on Earth, in stars, and in the universe as a whole is related to the binding energy of nuclei and has implications for the continued expansion of the universe.

Problem-Solving Strategies

For Reaction And Binding Energies and Activity Calculations in Nuclear Physics

1. *Identify exactly what needs to be determined in the problem (identify the unknowns)*. This will allow you to decide whether the energy of a decay or nuclear reaction is involved, for example, or whether the problem is primarily concerned with activity (rate of decay).
2. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).*
3. *For reaction and binding-energy problems, we use atomic rather than nuclear masses.* Since the masses of neutral atoms are used, you must count the number of electrons involved. If these do not balance (such as in β^+ decay), then an energy adjustment of 0.511 MeV per electron must be made. Also note that atomic masses may not be given in a problem; they can be found in tables.
4. *For problems involving activity, the relationship of activity to half-life, and the number of nuclei given in the equation* $R = \frac{0.693N}{t_{1/2}}$ *can be very useful.* Owing to the fact that number of nuclei is involved, you will also need to be familiar with moles and Avogadro's number.
5. *Perform the desired calculation; keep careful track of plus and minus signs as well as powers of 10.*
6. *Check the answer to see if it is reasonable: Does it make sense?* Compare your results with worked examples and other information in the text. (Heeding the advice in Step 5 will also help you to be certain of your result.) You must understand the problem conceptually to be able to determine whether the numerical result is reasonable.

PhET Explorations: Nuclear Fission

Start a chain reaction, or introduce non-radioactive isotopes to prevent one. Control energy production in a nuclear reactor!

PhET Interactive Simulation

Figure 31.29 Nuclear Fission (http://cnx.org/content/m42640/1.5/nuclear-fission_en.jar)

31.7 Tunneling

Protons and neutrons are *bound* inside nuclei, that means energy must be supplied to break them away. The situation is analogous to a marble in a bowl that can roll around but lacks the energy to get over the rim. It is bound inside the bowl (see **Figure 31.30**). If the marble could get over the rim, it would gain kinetic energy by rolling down outside. However classically, if the marble does not have enough kinetic energy to get over the rim, it remains forever trapped in its well.

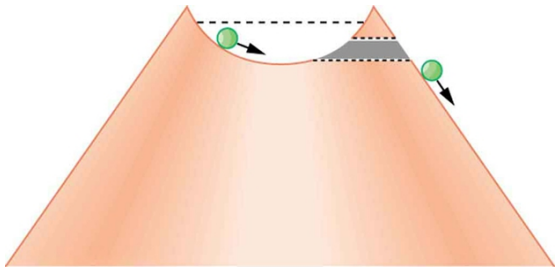


Figure 31.30 The marble in this semicircular bowl at the top of a volcano has enough kinetic energy to get to the altitude of the dashed line, but not enough to get over the rim, so that it is trapped forever. If it could find a tunnel through the barrier, it would escape, roll downhill, and gain kinetic energy.

In a nucleus, the attractive nuclear potential is analogous to the bowl at the top of a volcano (where the “volcano” refers only to the shape). Protons and neutrons have kinetic energy, but it is about 8 MeV less than that needed to get out (see **Figure 31.31**). That is, they are bound by an average of 8 MeV per nucleon. The slope of the hill outside the bowl is analogous to the repulsive Coulomb potential for a nucleus, such as for an α particle outside a positive nucleus. In α decay, two protons and two neutrons spontaneously break away as a ^4He unit. Yet the protons and neutrons do not have enough kinetic energy to get over the rim. So how does the α particle get out?

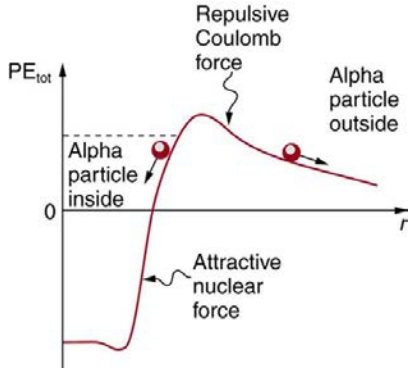


Figure 31.31 Nucleons within an atomic nucleus are bound or trapped by the attractive nuclear force, as shown in this simplified potential energy curve. An α particle outside the range of the nuclear force feels the repulsive Coulomb force. The α particle inside the nucleus does not have enough kinetic energy to get over the rim, yet it does manage to get out by quantum mechanical tunneling.

The answer was supplied in 1928 by the Russian physicist George Gamow (1904–1968). The α particle tunnels through a region of space it is forbidden to be in, and it comes out of the side of the nucleus. Like an electron making a transition between orbits around an atom, it travels from one point to another without ever having been in between. **Figure 31.32** indicates how this works. The wave function of a quantum mechanical particle varies smoothly, going from within an atomic nucleus (on one side of a potential energy barrier) to outside the nucleus (on the other side of the potential energy barrier). Inside the barrier, the wave function does not become zero but decreases exponentially, and we do not observe the particle inside the barrier. The probability of finding a particle is related to the square of its wave function, and so there is a small probability of finding the particle outside the barrier, which implies that the particle can tunnel through the barrier. This process is called **barrier penetration** or **quantum mechanical tunneling**. This concept was developed in theory by J. Robert Oppenheimer (who led the development of the first nuclear bombs during World War II) and was used by Gamow and others to describe α decay.

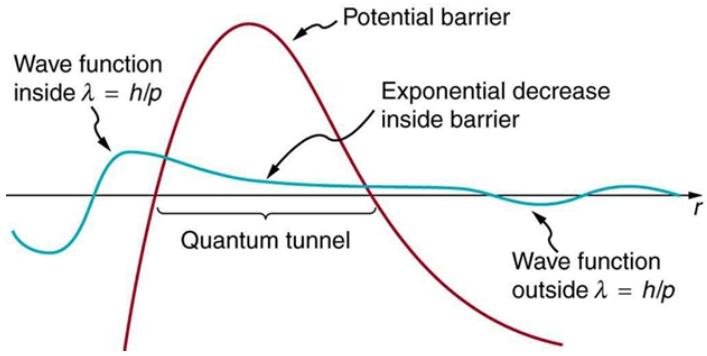


Figure 31.32 The wave function representing a quantum mechanical particle must vary smoothly, going from within the nucleus (to the left of the barrier) to outside the nucleus (to the right of the barrier). Inside the barrier, the wave function does not abruptly become zero; rather, it decreases exponentially. Outside the barrier, the wave function is small but finite, and there it smoothly becomes sinusoidal. Owing to the fact that there is a small probability of finding the particle outside the barrier, the particle can tunnel through the barrier.

Good ideas explain more than one thing. In addition to qualitatively explaining how the four nucleons in an α particle can get out of the nucleus, the detailed theory also explains quantitatively the half-life of various nuclei that undergo α decay. This description is what Gamow and others devised, and it works for α decay half-lives that vary by 17 orders of magnitude. Experiments have shown that the more energetic the α decay of a particular nuclide is, the shorter is its half-life. **Tunneling** explains this in the following manner: For the decay to be more energetic, the nucleons must have more energy in the nucleus and should be able to ascend a little closer to the rim. The barrier is therefore not as thick for more energetic decay, and the exponential decrease of the wave function inside the barrier is not as great. Thus the probability of finding the particle outside the barrier is greater, and the half-life is shorter.

Tunneling as an effect also occurs in quantum mechanical systems other than nuclei. Electrons trapped in solids can tunnel from one object to another if the barrier between the objects is thin enough. The process is the same in principle as described for α decay. It is far more likely for a thin barrier than a thick one. Scanning tunneling electron microscopes function on this principle. The current of electrons that travels between a probe and a sample tunnels through a barrier and is very sensitive to its thickness, allowing detection of individual atoms as shown in **Figure 31.33**.

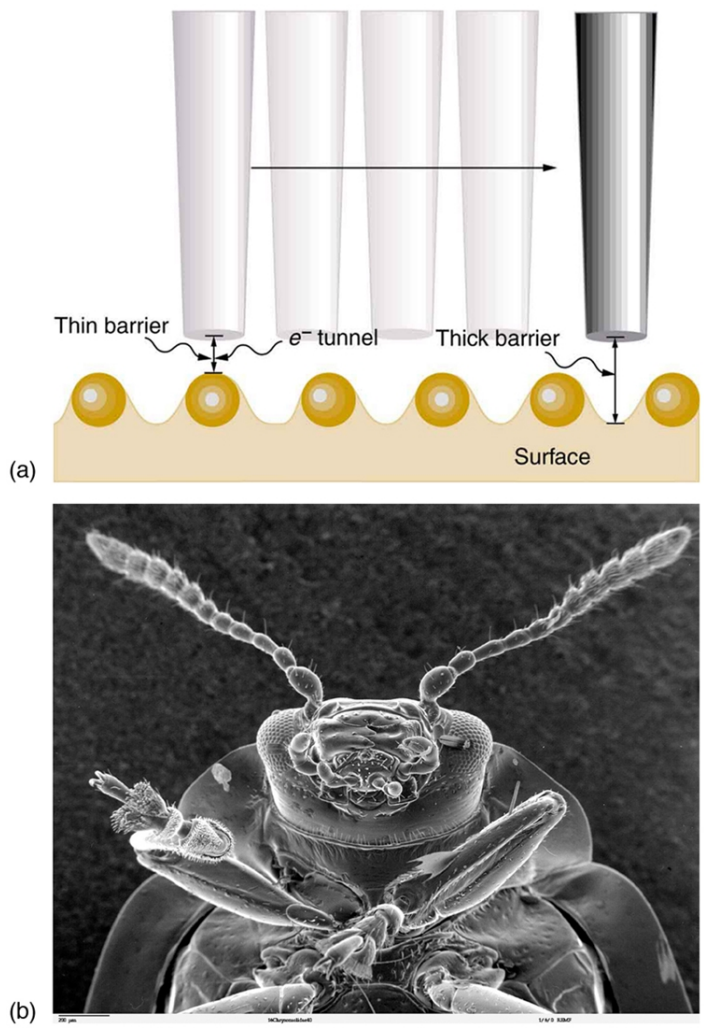


Figure 31.33 (a) A scanning tunneling electron microscope can detect extremely small variations in dimensions, such as individual atoms. Electrons tunnel quantum mechanically between the probe and the sample. The probability of tunneling is extremely sensitive to barrier thickness, so that the electron current is a sensitive indicator of surface features. (b) Head and mouthparts of *Coleoptera Chrysomelidea* as seen through an electron microscope (credit: Louisa Howard, Dartmouth College)

PhET Explorations: Quantum Tunneling and Wave Packets

Watch quantum "particles" tunnel through barriers. Explore the properties of the wave functions that describe these particles.

PhET Interactive Simulation

Figure 31.34 Quantum Tunneling and Wave Packets (http://cnx.org/content/m42644/1.5/quantum-tunneling_en.jar)

Glossary

activity: the rate of decay for radioactive nuclides

alpha decay: type of radioactive decay in which an atomic nucleus emits an alpha particle

alpha rays: one of the types of rays emitted from the nucleus of an atom

antielectron: another term for positron

antimatter: composed of antiparticles

atomic mass: the total mass of the protons, neutrons, and electrons in a single atom

atomic number: number of protons in a nucleus

barrier penetration: quantum mechanical effect whereby a particle has a nonzero probability to cross through a potential energy barrier despite not having sufficient energy to pass over the barrier; also called quantum mechanical tunneling

becquerel: SI unit for rate of decay of a radioactive material

beta decay: type of radioactive decay in which an atomic nucleus emits a beta particle

beta rays: one of the types of rays emitted from the nucleus of an atom

binding energy per nucleon: the binding energy calculated per nucleon; it reveals the details of the nuclear force—larger the BE/A, the more stable the nucleus

binding energy: the energy needed to separate nucleus into individual protons and neutrons

carbon-14 dating: a radioactive dating technique based on the radioactivity of carbon-14

chart of the nuclides: a table comprising stable and unstable nuclei

curie: the activity of 1g of $^{226}\mathrm{Ra}$, equal to $3.70\times10^{10}\ \mathrm{Bq}$

daughter: the nucleus obtained when parent nucleus decays and produces another nucleus following the rules and the conservation laws

decay constant: quantity that is inversely proportional to the half-life and that is used in equation for number of nuclei as a function of time

decay equation: the equation to find out how much of a radioactive material is left after a given period of time

decay series: process whereby subsequent nuclides decay until a stable nuclide is produced

decay: the process by which an atomic nucleus of an unstable atom loses mass and energy by emitting ionizing particles

electron capture equation: equation representing the electron capture

electron capture: the process in which a proton-rich nuclide absorbs an inner atomic electron and simultaneously emits a neutrino

electron's antineutrino: antiparticle of electron's neutrino

electron's neutrino: a subatomic elementary particle which has no net electric charge

Geiger tube: a very common radiation detector that usually gives an audio output

gamma decay: type of radioactive decay in which an atomic nucleus emits a gamma particle

gamma rays: one of the types of rays emitted from the nucleus of an atom

half-life: the time in which there is a 50% chance that a nucleus will decay

ionizing radiation: radiation (whether nuclear in origin or not) that produces ionization whether nuclear in origin or not

isotopes: nuclei having the same Z and different N s

magic numbers: a number that indicates a shell structure for the nucleus in which closed shells are more stable

mass number: number of nucleons in a nucleus

neutrino: an electrically neutral, weakly interacting elementary subatomic particle

neutron: a neutral particle that is found in a nucleus

nuclear radiation: rays that originate in the nuclei of atoms, the first examples of which were discovered by Becquerel

nuclear reaction energy: the energy created in a nuclear reaction

nucleons: the particles found inside nuclei

nucleus: a region consisting of protons and neutrons at the center of an atom

nuclide: a type of atom whose nucleus has specific numbers of protons and neutrons

parent: the original state of nucleus before decay

photomultiplier: a device that converts light into electrical signals

positron decay: type of beta decay in which a proton is converted to a neutron, releasing a positron and a neutrino

positron: the particle that results from positive beta decay; also known as an antielectron

protons: the positively charged nucleons found in a nucleus

quantum mechanical tunneling: quantum mechanical effect whereby a particle has a nonzero probability to cross through a potential energy barrier despite not having sufficient energy to pass over the barrier; also called barrier penetration

radiation detector: a device that is used to detect and track the radiation from a radioactive reaction

radioactive dating: an application of radioactive decay in which the age of a material is determined by the amount of radioactivity of a particular type that occurs

radioactive: a substance or object that emits nuclear radiation

radioactivity: the emission of rays from the nuclei of atoms

radius of a nucleus: the radius of a nucleus is $r = r_0 A^{1/3}$

range of radiation: the distance that the radiation can travel through a material

rate of decay: the number of radioactive events per unit time

scintillators: a radiation detection method that records light produced when radiation interacts with materials

solid-state radiation detectors: semiconductors fabricated to directly convert incident radiation into electrical current

tunneling: a quantum mechanical process of potential energy barrier penetration

Section Summary

31.1 Nuclear Radioactivity

- Some nuclei are radioactive—they spontaneously decay destroying some part of their mass and emitting energetic rays, a process called nuclear radioactivity.
- Nuclear radiation, like x rays, is ionizing radiation, because energy sufficient to ionize matter is emitted in each decay.
- The range (or distance traveled in a material) of ionizing radiation is directly related to the charge of the emitted particle and its energy, with greater-charge and lower-energy particles having the shortest ranges.
- Radiation detectors are based directly or indirectly upon the ionization created by radiation, as are the effects of radiation on living and inert materials.

31.2 Radiation Detection and Detectors

- Radiation detectors are based directly or indirectly upon the ionization created by radiation, as are the effects of radiation on living and inert materials.

31.3 Substructure of the Nucleus

- Two particles, both called nucleons, are found inside nuclei. The two types of nucleons are protons and neutrons; they are very similar, except that the proton is positively charged while the neutron is neutral. Some of their characteristics are given in **Table 31.2** and compared with those of the electron. A mass unit convenient to atomic and nuclear processes is the unified atomic mass unit (u), defined to be

$$1\text{ u} = 1.6605\times10^{-27}\text{ kg} = 931.46\text{ MeV}/c^2.$$

- A nuclide is a specific combination of protons and neutrons, denoted by
$$ {}^{A}_{Z}\mathrm{X}_{N} \text{ or simply } {}^{A}\mathrm{X}, $$
Z is the number of protons or atomic number, X is the symbol for the element, N is the number of neutrons, and A is the mass number or the total number of protons and neutrons,
$$ A = N + Z. $$
- Nuclides having the same Z but different N are isotopes of the same element.
- The radius of a nucleus, r, is approximately
$$ r = r_0 A^{1/3}, $$
where $r_0 = 1.2 \text{ fm}$. Nuclear volumes are proportional to A. There are two nuclear forces, the weak and the strong. Systematics in nuclear stability seen on the chart of the nuclides indicate that there are shell closures in nuclei for values of Z and N equal to the magic numbers, which correspond to highly stable nuclei.

31.4 Nuclear Decay and Conservation Laws

- When a parent nucleus decays, it produces a daughter nucleus following rules and conservation laws. There are three major types of nuclear decay, called alpha (α), beta (β), and gamma (γ). The α decay equation is
$$ {}^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y}_{N-2} + {}^{4}_{2}\mathrm{He}_{2}. $$
- Nuclear decay releases an amount of energy E related to the mass destroyed Δm by
$$ E = (\Delta m)c^2. $$
- There are three forms of beta decay. The β^- decay equation is
$$ {}^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A}_{Z+1}\mathrm{Y}_{N-1} + \beta^- + \bar{\nu}_e. $$
- The β^+ decay equation is
$$ {}^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A}_{Z-1}\mathrm{Y}_{N+1} + \beta^+ + \nu_e. $$
- The electron capture equation is
$$ {}^{A}_{Z}\mathrm{X}_{N} + e^- \rightarrow {}^{A}_{Z-1}\mathrm{Y}_{N+1} + \nu_e. $$
- β^- is an electron, β^+ is an antielectron or positron, ν_e represents an electron's neutrino, and $\bar{\nu}_e$ is an electron's antineutrino. In addition to all previously known conservation laws, two new ones arise— conservation of electron family number and conservation of the total number of nucleons. The γ decay equation is
$$ \mathrm{X}^*_N \rightarrow \mathrm{X}_N + \gamma_1 + \gamma_2 + \cdots $$
γ is a high-energy photon originating in a nucleus.

31.5 Half-Life and Activity

- Half-life $t_{1/2}$ is the time in which there is a 50% chance that a nucleus will decay. The number of nuclei N as a function of time is
$$ N = N_0 e^{-\lambda t}, $$
where N_0 is the number present at $t = 0$, and λ is the decay constant, related to the half-life by
$$ \lambda = \frac{0.693}{t_{1/2}}. $$
- One of the applications of radioactive decay is radioactive dating, in which the age of a material is determined by the amount of radioactive decay that occurs. The rate of decay is called the activity R:
$$ R = \frac{\Delta N}{\Delta t}. $$
- The SI unit for R is the becquerel (Bq), defined by
$$ 1 \text{ Bq} = 1 \text{ decay/s}. $$
- R is also expressed in terms of curies (Ci), where
$$ 1 \text{ Ci} = 3.70 \times 10^{10} \text{ Bq}. $$
- The activity R of a source is related to N and $t_{1/2}$ by
$$ R = \frac{0.693N}{t_{1/2}}. $$
- Since N has an exponential behavior as in the equation $N = N_0 e^{-\lambda t}$, the activity also has an exponential behavior, given by
$$ R = R_0 e^{-\lambda t}, $$
where R_0 is the activity at $t = 0$.

31.6 Binding Energy

- The binding energy (BE) of a nucleus is the energy needed to separate it into individual protons and neutrons. In terms of atomic masses,

$$\text{BE} = \{[Zm(^1\text{H}) + Nm_n] - m(^A\text{X})\}c^2,$$

where $m\left(^1\text{H}\right)$ is the mass of a hydrogen atom, $m\left(^A\text{X}\right)$ is the atomic mass of the nuclide, and m_n is the mass of a neutron. Patterns in the binding energy per nucleon, BE/A, reveal details of the nuclear force. The larger the BE/A, the more stable the nucleus.

31.7 Tunneling

- Tunneling is a quantum mechanical process of potential energy barrier penetration. The concept was first applied to explain α decay, but tunneling is found to occur in other quantum mechanical systems.

Conceptual Questions

31.1 Nuclear Radioactivity

1. Suppose the range for $5.0\ \text{MeV}\alpha$ ray is known to be 2.0 mm in a certain material. Does this mean that every $5.0\ \text{MeV}\alpha$ a ray that strikes this material travels 2.0 mm, or does the range have an average value with some statistical fluctuations in the distances traveled? Explain.

2. What is the difference between γ rays and characteristic x rays? Is either necessarily more energetic than the other? Which can be the most energetic?

3. Ionizing radiation interacts with matter by scattering from electrons and nuclei in the substance. Based on the law of conservation of momentum and energy, explain why electrons tend to absorb more energy than nuclei in these interactions.

4. What characteristics of radioactivity show it to be nuclear in origin and not atomic?

5. What is the source of the energy emitted in radioactive decay? Identify an earlier conservation law, and describe how it was modified to take such processes into account.

6. Consider **Figure 31.3**. If an electric field is substituted for the magnetic field with positive charge instead of the north pole and negative charge instead of the south pole, in which directions will the α, β, and γ rays bend?

7. Explain how an α particle can have a larger range in air than a β particle with the same energy in lead.

8. Arrange the following according to their ability to act as radiation shields, with the best first and worst last. Explain your ordering in terms of how radiation loses its energy in matter.

(a) A solid material with low density composed of low-mass atoms.

(b) A gas composed of high-mass atoms.

(c) A gas composed of low-mass atoms.

(d) A solid with high density composed of high-mass atoms.

9. Often, when people have to work around radioactive materials spills, we see them wearing white coveralls (usually a plastic material). What types of radiation (if any) do you think these suits protect the worker from, and how?

31.2 Radiation Detection and Detectors

10. Is it possible for light emitted by a scintillator to be too low in frequency to be used in a photomultiplier tube? Explain.

31.3 Substructure of the Nucleus

11. The weak and strong nuclear forces are basic to the structure of matter. Why we do not experience them directly?

12. Define and make clear distinctions between the terms neutron, nucleon, nucleus, nuclide, and neutrino.

13. What are isotopes? Why do different isotopes of the same element have similar chemistries?

31.4 Nuclear Decay and Conservation Laws

14. Star Trek fans have often heard the term “antimatter drive.” Describe how you could use a magnetic field to trap antimatter, such as produced by nuclear decay, and later combine it with matter to produce energy. Be specific about the type of antimatter, the need for vacuum storage, and the fraction of matter converted into energy.

15. What conservation law requires an electron’s neutrino to be produced in electron capture? Note that the electron no longer exists after it is captured by the nucleus.

16. Neutrinos are experimentally determined to have an extremely small mass. Huge numbers of neutrinos are created in a supernova at the same time as massive amounts of light are first produced. When the 1987A supernova occurred in the Large Magellanic Cloud, visible primarily in the Southern Hemisphere and some 100,000 light-years away from Earth, neutrinos from the explosion were observed at about the same time as the light from the blast. How could the relative arrival times of neutrinos and light be used to place limits on the mass of neutrinos?

17. What do the three types of beta decay have in common that is distinctly different from alpha decay?

31.5 Half-Life and Activity

18. In a 3×10^9 -year-old rock that originally contained some ^{238}U , which has a half-life of 4.5×10^9 years, we expect to find some ^{238}U remaining in it. Why are ^{226}Ra , ^{222}Rn , and ^{210}Po also found in such a rock, even though they have much shorter half-lives (1600 years, 3.8 days, and 138 days, respectively)?

19. Does the number of radioactive nuclei in a sample decrease to *exactly* half its original value in one half-life? Explain in terms of the statistical nature of radioactive decay.

20. Radioactivity depends on the nucleus and not the atom or its chemical state. Why, then, is one kilogram of uranium more radioactive than one kilogram of uranium hexafluoride?

21. Explain how a bound system can have less mass than its components. Why is this not observed classically, say for a building made of bricks?

22. Spontaneous radioactive decay occurs only when the decay products have less mass than the parent, and it tends to produce a daughter that is more stable than the parent. Explain how this is related to the fact that more tightly bound nuclei are more stable. (Consider the binding energy per nucleon.)

23. To obtain the most precise value of BE from the equation $\text{BE}=\left[ZM\left(^1\text{H}\right)+Nm_n\right]c^2-m\left(^A\text{X}\right)c^2$, we should take into account the binding energy of the electrons in the neutral atoms. Will doing this produce a larger or smaller value for BE? Why is this effect usually negligible?

24. How does the finite range of the nuclear force relate to the fact that BE/A is greatest for A near 60?

31.6 Binding Energy

25. Why is the number of neutrons greater than the number of protons in stable nuclei having A greater than about 40, and why is this effect more pronounced for the heaviest nuclei?

31.7 Tunneling

26. A physics student caught breaking conservation laws is imprisoned. She leans against the cell wall hoping to tunnel out quantum mechanically. Explain why her chances are negligible. (This is so in any classical situation.)

27. When a nucleus α decays, does the α particle move continuously from inside the nucleus to outside? That is, does it travel each point along an imaginary line from inside to out? Explain.

Problems & Exercises

31.2 Radiation Detection and Detectors

28. The energy of 30.0 eV is required to ionize a molecule of the gas inside a Geiger tube, thereby producing an ion pair. Suppose a particle of ionizing radiation deposits 0.500 MeV of energy in this Geiger tube. What maximum number of ion pairs can it create?

29. A particle of ionizing radiation creates 4000 ion pairs in the gas inside a Geiger tube as it passes through. What minimum energy was deposited, if 30.0 eV is required to create each ion pair?

30. (a) Repeat **Exercise 31.29**, and convert the energy to joules or calories. (b) If all of this energy is converted to thermal energy in the gas, what is its temperature increase, assuming $50.0\ \text{cm}^3$ of ideal gas at 0.250-atm pressure? (The small answer is consistent with the fact that the energy is large on a quantum mechanical scale but small on a macroscopic scale.)

31. Suppose a particle of ionizing radiation deposits 1.0 MeV in the gas of a Geiger tube, all of which goes to creating ion pairs. Each ion pair requires 30.0 eV of energy. (a) The applied voltage sweeps the ions out of the gas in $1.00\ \mu\text{s}$. What is the current? (b) This current is smaller than the actual current since the applied voltage in the Geiger tube accelerates the separated ions, which then create other ion pairs in subsequent collisions. What is the current if this last effect multiplies the number of ion pairs by 900?

31.3 Substructure of the Nucleus

32. Verify that a $2.3\times10^{17}\ \text{kg}$ mass of water at normal density would make a cube 60 km on a side, as claimed in **Example 31.1**. (This mass at nuclear density would make a cube 1.0 m on a side.)

33. Find the length of a side of a cube having a mass of 1.0 kg and the density of nuclear matter, taking this to be $2.3\times10^{17}\ \text{kg/m}^3$.

34. What is the radius of an α particle?

35. Find the radius of a ^{238}Pu nucleus. ^{238}Pu is a manufactured nuclide that is used as a power source on some space probes.

36. (a) Calculate the radius of ^{58}Ni, one of the most tightly bound stable nuclei.

(b) What is the ratio of the radius of ^{58}Ni to that of ^{258}Ha, one of the largest nuclei ever made? Note that the radius of the largest nucleus is still much smaller than the size of an atom.

37. The unified atomic mass unit is defined to be $1\ \text{u} = 1.6605\times10^{-27}\ \text{kg}$. Verify that this amount of mass converted to energy yields 931.5 MeV. Note that you must use four-digit or better values for c and $|q_e|$.

38. What is the ratio of the velocity of a β particle to that of an α particle, if they have the same nonrelativistic kinetic energy?

39. If a 1.50-cm-thick piece of lead can absorb 90.0% of the γ rays from a radioactive source, how many centimeters of lead are needed to absorb all but 0.100% of the γ rays?

40. The detail observable using a probe is limited by its wavelength. Calculate the energy of a γ-ray photon that has a wavelength of $1\times10^{-16}\ \text{m}$, small enough to detect details about one-tenth the size of a nucleon. Note that a photon having this energy is difficult to produce and interacts poorly with the nucleus, limiting the practicability of this probe.

41. (a) Show that if you assume the average nucleus is spherical with a radius $r = r_0A^{1/3}$, and with a mass of A u, then its density is independent of A.

(b) Calculate that density in u/fm^3 and kg/m^3, and compare your results with those found in **Example 31.1** for ^{56}Fe.

42. What is the ratio of the velocity of a 5.00-MeV β ray to that of an α particle with the same kinetic energy? This should confirm that βs travel much faster than αs even when relativity is taken into consideration. (See also **Exercise 31.38**.)

43. (a) What is the kinetic energy in MeV of a β ray that is traveling at $0.998c$? This gives some idea of how energetic a β ray must be to travel at nearly the same speed as a γ ray. (b) What is the velocity of the γ ray relative to the β ray?

31.4 Nuclear Decay and Conservation Laws

In the following eight problems, write the complete decay equation for the given nuclide in the complete $^A_Z\text{X}_N$ notation. Refer to the periodic table for values of Z.

44. β^- decay of ^3H (tritium), a manufactured isotope of hydrogen used in some digital watch displays, and manufactured primarily for use in hydrogen bombs.

45. β^- decay of ^{40}K, a naturally occurring rare isotope of potassium responsible for some of our exposure to background radiation.

46. β^+ decay of ^{50}Mn.

47. β^+ decay of ^{52}Fe.

48. Electron capture by ^7Be.

49. Electron capture by ^{106}In.

50. α decay of ^{210}Po, the isotope of polonium in the decay series of ^{238}U that was discovered by the Curies. A favorite isotope in physics labs, since it has a short half-life and decays to a stable nuclide.

51. α decay of ^{226}Ra, another isotope in the decay series of ^{238}U, first recognized as a new element by the Curies. Poses special problems because its daughter is a radioactive noble gas.

In the following four problems, identify the parent nuclide and write the complete decay equation in the $^A_Z\text{X}_N$ notation. Refer to the periodic table for values of Z.

52. β^- decay producing ^{137}Ba. The parent nuclide is a major waste product of reactors and has chemistry similar to potassium and sodium, resulting in its concentration in your cells if ingested.

53. β^- decay producing ^{90}Y. The parent nuclide is a major waste product of reactors and has chemistry similar to calcium, so that it is concentrated in bones if ingested (^{90}Y is also radioactive.)

54. α decay producing ^{228}Ra. The parent nuclide is nearly 100% of the natural element and is found in gas lantern mantles and in metal alloys used in jets (^{228}Ra is also radioactive).

55. α decay producing ^{208}Pb. The parent nuclide is in the decay series produced by ^{232}Th, the only naturally occurring isotope of thorium.

56. When an electron and positron annihilate, both their masses are destroyed, creating two equal energy photons to preserve momentum.

(a) Confirm that the annihilation equation $e^+ + e^- \rightarrow \gamma + \gamma$ conserves charge, electron family number, and total number of nucleons. To do this, identify the values of each before and after the annihilation. (b) Find the energy of each γ ray, assuming the electron and positron are initially nearly at rest. (c) Explain why the two γ rays travel in exactly opposite directions if the center of mass of the electron-positron system is initially at rest.

57. Confirm that charge, electron family number, and the total number of nucleons are all conserved by the rule for α decay given in the equation ${}^{A}_{Z}\text{X}_N \rightarrow {}^{A-4}_{Z-2}\text{Y}_{N-2} + {}^{4}_{2}\text{He}_2$. To do this, identify the values of each before and after the decay.

58. Confirm that charge, electron family number, and the total number of nucleons are all conserved by the rule for β^- decay given in the equation ${}^{A}_{Z}\text{X}_N \rightarrow {}^{A}_{Z+1}\text{Y}_{N-1} + \beta^- + \bar{\nu}_e$. To do this, identify the values of each before and after the decay.

59. Confirm that charge, electron family number, and the total number of nucleons are all conserved by the rule for β^- decay given in the equation ${}^{A}_{Z}\text{X}_N \rightarrow {}^{A}_{Z-1}\text{Y}_{N-1} + \beta^- + \nu_e$. To do this, identify the values of each before and after the decay.

60. Confirm that charge, electron family number, and the total number of nucleons are all conserved by the rule for electron capture given in the equation ${}^{A}_{Z}\text{X}_N + e^- \rightarrow {}^{A}_{Z-1}\text{Y}_{N+1} + \nu_e$. To do this, identify the values of each before and after the capture.

61. A rare decay mode has been observed in which ^{222}Ra emits a ^{14}C nucleus. (a) The decay equation is $^{222}\text{Ra} \rightarrow {}^{A}\text{X} + {}^{14}\text{C}$. Identify the nuclide ^{A}X. (b) Find the energy emitted in the decay. The mass of ^{222}Ra is 222.015353 u.

62. (a) Write the complete α decay equation for ^{226}Ra.

(b) Find the energy released in the decay.

63. (a) Write the complete α decay equation for ^{249}Cf.

(b) Find the energy released in the decay.

64. (a) Write the complete β^- decay equation for the neutron. (b) Find the energy released in the decay.

65. (a) Write the complete β^- decay equation for ^{90}Sr, a major waste product of nuclear reactors. (b) Find the energy released in the decay.

66. Calculate the energy released in the β^+ decay of ^{22}Na, the equation for which is given in the text. The masses of ^{22}Na and ^{22}Ne are 21.994434 and 21.991383 u, respectively.

67. (a) Write the complete β^+ decay equation for ^{11}C.

(b) Calculate the energy released in the decay. The masses of ^{11}C and ^{11}B are 11.011433 and 11.009305 u, respectively.

68. (a) Calculate the energy released in the α decay of ^{238}U.

(b) What fraction of the mass of a single ^{238}U is destroyed in the decay? The mass of ^{234}Th is 234.043593 u.

(c) Although the fractional mass loss is large for a single nucleus, it is difficult to observe for an entire macroscopic sample of uranium. Why is this?

69. (a) Write the complete reaction equation for electron capture by ^{7}Be.

(b) Calculate the energy released.

70. (a) Write the complete reaction equation for electron capture by ^{15}O.

(b) Calculate the energy released.

31.5 Half-Life and Activity

Data from the appendices and the periodic table may be needed for these problems.

71. An old campfire is uncovered during an archaeological dig. Its charcoal is found to contain less than 1/1000 the normal amount of ^{14}C. Estimate the minimum age of the charcoal, noting that $2^{10} = 1024$.

72. A ^{60}Co source is labeled 4.00 mCi, but its present activity is found to be 1.85×10^7 Bq. (a) What is the present activity in mCi? (b) How long ago did it actually have a 4.00-mCi activity?

73. (a) Calculate the activity R in curies of 1.00 g of ^{226}Ra. (b) Discuss why your answer is not exactly 1.00 Ci, given that the curie was originally supposed to be exactly the activity of a gram of radium.

74. Show that the activity of the ^{14}C in 1.00 g of ^{12}C found in living tissue is 0.250 Bq.

75. Mantles for gas lanterns contain thorium, because it forms an oxide that can survive being heated to incandescence for long periods of time. Natural thorium is almost 100% ^{232}Th, with a half-life of 1.405×10^{10} y. If an average lantern mantle contains 300 mg of thorium, what is its activity?

76. Cow's milk produced near nuclear reactors can be tested for as little as 1.00 pCi of ^{131}I per liter, to check for possible reactor leakage. What mass of ^{131}I has this activity?

77. (a) Natural potassium contains ^{40}K, which has a half-life of 1.277×10^9 y. What mass of ^{40}K in a person would have a decay rate of 4140 Bq? (b) What is the fraction of ^{40}K in natural potassium, given that the person has 140 g in his body? (These numbers are typical for a 70-kg adult.)

78. There is more than one isotope of natural uranium. If a researcher isolates 1.00 mg of the relatively scarce ^{235}U and finds this mass to have an activity of 80.0 Bq, what is its half-life in years?

79. ^{50}V has one of the longest known radioactive half-lives. In a difficult experiment, a researcher found that the activity of 1.00 kg of ^{50}V is 1.75 Bq. What is the half-life in years?

80. You can sometimes find deep red crystal vases in antique stores, called uranium glass because their color was produced by doping the glass with uranium. Look up the natural isotopes of uranium and their

half-lives, and calculate the activity of such a vase assuming it has 2.00 g of uranium in it. Neglect the activity of any daughter nuclides.

81. A tree falls in a forest. How many years must pass before the ^{14}C activity in 1.00 g of the tree's carbon drops to 1.00 decay per hour?

82. What fraction of the ^{40}K that was on Earth when it formed 4.5×10^{9} years ago is left today?

83. A 5000-Ci ^{60}Co source used for cancer therapy is considered too weak to be useful when its activity falls to 3500 Ci. How long after its manufacture does this happen?

84. Natural uranium is 0.7200% ^{235}U and 99.27% ^{238}U. What were the percentages of ^{235}U and ^{238}U in natural uranium when Earth formed 4.5×10^{9} years ago?

85. The β^{-} particles emitted in the decay of ^{3}H (tritium) interact with matter to create light in a glow-in-the-dark exit sign. At the time of manufacture, such a sign contains 15.0 Ci of ^{3}H. (a) What is the mass of the tritium? (b) What is its activity 5.00 y after manufacture?

86. World War II aircraft had instruments with glowing radium-painted dials (see **Figure 31.2**). The activity of one such instrument was 1.0×10^{5} Bq when new. (a) What mass of ^{226}Ra was present? (b) After some years, the phosphors on the dials deteriorated chemically, but the radium did not escape. What is the activity of this instrument 57.0 years after it was made?

87. (a) The ^{210}Po source used in a physics laboratory is labeled as having an activity of $1.0\ \mu\text{Ci}$ on the date it was prepared. A student measures the radioactivity of this source with a Geiger counter and observes 1500 counts per minute. She notices that the source was prepared 120 days before her lab. What fraction of the decays is she observing with her apparatus? (b) Identify some of the reasons that only a fraction of the α s emitted are observed by the detector.

88. Armor-piercing shells with depleted uranium cores are fired by aircraft at tanks. (The high density of the uranium makes them effective.) The uranium is called depleted because it has had its ^{235}U removed for reactor use and is nearly pure ^{238}U. Depleted uranium has been erroneously called non-radioactive. To demonstrate that this is wrong: (a) Calculate the activity of 60.0 g of pure ^{238}U. (b) Calculate the activity of 60.0 g of natural uranium, neglecting the ^{234}U and all daughter nuclides.

89. The ceramic glaze on a red-orange Fiestaware plate is U_2O_3 and contains 50.0 grams of ^{238}U, but very little ^{235}U. (a) What is the activity of the plate? (b) Calculate the total energy that will be released by the ^{238}U decay. (c) If energy is worth 12.0 cents per $\text{kW}\cdot\text{h}$, what is the monetary value of the energy emitted? (These plates went out of production some 30 years ago, but are still available as collectibles.)

90. Large amounts of depleted uranium (^{238}U) are available as a by-product of uranium processing for reactor fuel and weapons. Uranium is very dense and makes good counter weights for aircraft. Suppose you have a 4000-kg block of ^{238}U. (a) Find its activity. (b) How many calories per day are generated by thermalization of the decay energy? (c) Do you think you could detect this as heat? Explain.

91. The *Galileo* space probe was launched on its long journey past several planets in 1989, with an ultimate goal of Jupiter. Its power source is 11.0 kg of ^{238}Pu, a by-product of nuclear weapons plutonium production. Electrical energy is generated thermoelectrically from the heat produced when the 5.59-MeV α particles emitted in each decay crash to a halt inside the plutonium and its shielding. The half-life of ^{238}Pu is 87.7 years. (a) What was the original activity of the ^{238}Pu in becquerel? (b) What power was emitted in kilowatts? (c) What power was emitted 12.0 y after launch? You may neglect any extra energy from daughter nuclides and any losses from escaping γ rays.

92. Construct Your Own Problem

Consider the generation of electricity by a radioactive isotope in a space probe, such as described in **Exercise 31.91**. Construct a problem in which you calculate the mass of a radioactive isotope you need in order to supply power for a long space flight. Among the things to consider are the isotope chosen, its half-life and decay energy, the power needs of the probe and the length of the flight.

93. Unreasonable Results

A nuclear physicist finds $1.0\ \mu\text{g}$ of ^{236}U in a piece of uranium ore and assumes it is primordial since its half-life is $2.3\times10^{7}\ \text{y}$. (a) Calculate the amount of ^{236}U that would had to have been on Earth when it formed $4.5\times10^{9}\ \text{y}$ ago for $1.0\ \mu\text{g}$ to be left today. (b) What is unreasonable about this result? (c) What assumption is responsible?

94. Unreasonable Results

(a) Repeat **Exercise 31.84** but include the 0.0055% natural abundance of ^{234}U with its $2.45\times10^{5}\ \text{y}$ half-life. (b) What is unreasonable about this result? (c) What assumption is responsible? (d) Where does the ^{234}U come from if it is not primordial?

95. Unreasonable Results

The manufacturer of a smoke alarm decides that the smallest current of α radiation he can detect is $1.00\ \mu\text{A}$. (a) Find the activity in curies of an α emitter that produces a $1.00\ \mu\text{A}$ current of α particles. (b) What is unreasonable about this result? (c) What assumption is responsible?

31.6 Binding Energy

96. ^{2}H is a loosely bound isotope of hydrogen. Called deuterium or heavy hydrogen, it is stable but relatively rare—it is 0.015% of natural hydrogen. Note that deuterium has $Z = N$, which should tend to make it more tightly bound, but both are odd numbers. Calculate BE/A, the binding energy per nucleon, for ^{2}H and compare it with the approximate value obtained from the graph in **Figure 31.27**.

97. ^{56}Fe is among the most tightly bound of all nuclides. It is more than 90% of natural iron. Note that ^{56}Fe has even numbers of both protons and neutrons. Calculate BE/A, the binding energy per nucleon, for ^{56}Fe and compare it with the approximate value obtained from the graph in **Figure 31.27**.

98. ^{209}Bi is the heaviest stable nuclide, and its BE/A is low compared with medium-mass nuclides. Calculate BE/A, the binding energy per nucleon, for ^{209}Bi and compare it with the approximate value obtained from the graph in **Figure 31.27**.

99. (a) Calculate BE/A for ^{235}U, the rarer of the two most common uranium isotopes. (b) Calculate BE/A for ^{238}U. (Most of uranium is

^{238}U .) Note that ^{238}U has even numbers of both protons and neutrons. Is the BE/A of ^{238}U significantly different from that of ^{235}U ?

100. (a) Calculate BE/A for ^{12}C . Stable and relatively tightly bound, this nuclide is most of natural carbon. (b) Calculate BE/A for ^{14}C . Is the difference in BE/A between ^{12}C and ^{14}C significant? One is stable and common, and the other is unstable and rare.

101. The fact that BE/A is greatest for A near 60 implies that the range of the nuclear force is about the diameter of such nuclides. (a) Calculate the diameter of an $A = 60$ nucleus. (b) Compare BE/A for ^{58}Ni and ^{90}Sr . The first is one of the most tightly bound nuclides, while the second is larger and less tightly bound.

102. The purpose of this problem is to show in three ways that the binding energy of the electron in a hydrogen atom is negligible compared with the masses of the proton and electron. (a) Calculate the mass equivalent in u of the 13.6-eV binding energy of an electron in a hydrogen atom, and compare this with the mass of the hydrogen atom obtained from **Appendix A**. (b) Subtract the mass of the proton given in **Table 31.2** from the mass of the hydrogen atom given in **Appendix A**. You will find the difference is equal to the electron's mass to three digits, implying the binding energy is small in comparison. (c) Take the ratio of the binding energy of the electron (13.6 eV) to the energy equivalent of the electron's mass (0.511 MeV). (d) Discuss how your answers confirm the stated purpose of this problem.

103. Unreasonable Results

A particle physicist discovers a neutral particle with a mass of 2.02733 u that he assumes is two neutrons bound together. (a) Find the binding energy. (b) What is unreasonable about this result? (c) What assumptions are unreasonable or inconsistent?

31.7 Tunneling

104. Derive an approximate relationship between the energy of α decay and half-life using the following data. It may be useful to graph the log of $t_{1/2}$ against E_α to find some straight-line relationship.

Table 31.3 Energy and Half-Life for α Decay

Nuclide	E_α (MeV)	$t_{1/2}$
^{216}Ra	9.5	0.18 μs
^{194}Po	7.0	0.7 s
^{240}Cm	6.4	27 d
^{226}Ra	4.91	1600 y
^{232}Th	4.1	1.4×10^{10} y

105. Integrated Concepts

A 2.00-T magnetic field is applied perpendicular to the path of charged particles in a bubble chamber. What is the radius of curvature of the path of a 10 MeV proton in this field? Neglect any slowing along its path.

106. (a) Write the decay equation for the α decay of ^{235}U . (b) What energy is released in this decay? The mass of the daughter nuclide is 231.036298 u. (c) Assuming the residual nucleus is formed in its ground state, how much energy goes to the α particle?

107. Unreasonable Results

The relatively scarce naturally occurring calcium isotope ^{48}Ca has a half-life of about 2×10^{16} y . (a) A small sample of this isotope is labeled as having an activity of 1.0 Ci. What is the mass of the ^{48}Ca in the sample? (b) What is unreasonable about this result? (c) What assumption is responsible?

108. Unreasonable Results

A physicist scatters γ rays from a substance and sees evidence of a nucleus 7.5×10^{-13} m in radius. (a) Find the atomic mass of such a nucleus. (b) What is unreasonable about this result? (c) What is unreasonable about the assumption?

109. Unreasonable Results

A frazzled theoretical physicist reckons that all conservation laws are obeyed in the decay of a proton into a neutron, positron, and neutrino (as in β^+ decay of a nucleus) and sends a paper to a journal to announce the reaction as a possible end of the universe due to the spontaneous decay of protons. (a) What energy is released in this decay? (b) What is unreasonable about this result? (c) What assumption is responsible?

110. Construct Your Own Problem

Consider the decay of radioactive substances in the Earth's interior. The energy emitted is converted to thermal energy that reaches the earth's surface and is radiated away into cold dark space. Construct a problem in which you estimate the activity in a cubic meter of earth rock? And then calculate the power generated. Calculate how much power must cross each square meter of the Earth's surface if the power is dissipated at the same rate as it is generated. Among the things to consider are the activity per cubic meter, the energy per decay, and the size of the Earth.